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HELPS TO

GEOMETRICAL DRAWING

PART II.

Projections, Practical Solid Geometry, Interpenetration
of solids and cast shadows.

Intended for the use of Students of the Engineering Colleges
and other Technical Schools in India, of the Survey
Schools in Bengal, Behar and Orissa
and for Art Students

BY

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other useful notes on Building constn. &c

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PREFACE

(SECOND EDITION)

The practical plane geometry is suited for school course and it has been treated in part I of this book. Part II contains projections, sections, developments of surfaces, isometric projection (a particular case of ordinary projection), descriptive geometry or, practical solid geometry, inter-penetration of solids and cast shadows and line of separation of light and shade

I have arranged the different subjects of practical solid geometry in such a way as to make them intelligible and easier for Indian students to understand without help from teachers. I have omitted the higher parts of descriptive geometry which are too obstruse for ordinary students. Some of the problems in this book have been compiled from other standard books on the subject and some new problems have been added which I found useful to the student during my long experience in the teaching of the subjects, I have tried to make the book concise and at the same time useful to the students. I shall be grateful to the teachers and other readers of this book if they will kindly point out any mistake or omission in the book. I shall also gratefully receive all suggestions for its improvement.

SIBPUR,
24th April, 1925.

C. K. BASU.

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CHAPTER I

PROJECTIONS.

Linear drawing deals with plane figures i.e. figures which have only two dimensions length and breadth. To represent solid figures with three dimensions (length, breadth and thickness) on a plane surface in such a way that they may be clearly understood in all their parts, there must be at least two drawings, one corresponds to the other, and the two drawings, are called the "projections of solid objects." The forms of projections change as the object varies in positions.

The difference between pure solid geometry and practical solid geometry is that the former deals with the geometrical relations which exist amongst points and lines of solid objects in space while the latter shows how to exhibit these relations by scale drawings which can be measured.

In solid geometry all objects are represented as they would appear traced or projected on two planes at right angles to each other, one being horizontal and the other vertical and which are named co-ordinate planes.

Projections of different points, of an object in order to represent it on the two co-ordinate planes are made on the imaginary supposition, that the eye is directly opposite each point at the same time, and is looking perpendicularly to the plane. This sort of projection is called the orthographic or orthogonal or parallel projection. It is impossible to see an object as it is represented by the orthographic projection as the eye cannot be directly opposite each point at the same time.

The lines drawn from the points in the object perpendicular to the two planes of projection are called the "projectors," the points in which these lines meet the two co-ordinate planes are called the "projections."

When the projections are found on the horizontal plane by looking vertically down over each point, the figure obtained by joining the points is called the *plan* or the horizontal projection, and when the figure is traced on the vertical plane by looking in front of each point, in a direction perpendicular to the vertical plane, it is called the *elevation* or the vertical projection.

What we actually see is the perspective projection of an object which is taken from one fixed point i. e. the eye and the lines drawn from the different parts of the object converge towards the eye considered as a fixed point. These lines are the rays of light from the object to the eye which form a sort of cone, the vertext of which is the eye. It is the natural representation of the object but not suited for scale drawing as the lines of the figure drawn do not retain their relative proportions.

Rules of projections :—

From the methods of looking perpendicularly to the plane it will be seen that :—

(1) The angles made by the objects with the horizontal plane will be represented on the V. P. and the angles with the vertical plane will be represented on the H. P. i. e. the angle made by an object with a co-ordinate plane is shown on the other plane.

(2). That an object for instance, a straight line is seen full size on a plane when it is parallel to the plane and will decrease in size in plan or elevation as the angles it makes with the H. P. or V. P. increase.

(3). That the length of the horizontal projector of a point from the ground line i. e. the line of intersection of the two co-ordinate planes shows the distance of the point from the vertical plane, and the length of the vertical projector its height from the horizontal plane.

(4). The line joining the plan and elevation of the same point is perpendicular to the ground line.

It is convenient to name the actual corners of an object or points in space by capital letters, their projections on the H. P. or plans, by the respective small letters and their elevations by small letters with a dash, as A is a point in space its plan is *a* and its elevation is a'. The ground line is represented by the letters XY. The lower points in plan always represent the front points in the elevation as the vertical plane is behind the object.

Full top view is seen in plan when an object is placed on the horizontal plane or held parallel to the H. P. Full front view is seen in elevation when the object is so placed that its front is parallel to the V. P. As long as the parallel position is not altered the view retains the same form but may be in different positions.

Simple projection :—It is somewhat easier for a student to comprehend the simple projections of solid figures than the projection of lines and planes as the eye is assisted by the solid appearance of the figure.

The simplest position of an object is when it is placed parallel to both the planes of projection i. e. the base is placed on the H. P. and a face or axis of the object is parallel to the V. P.

The next harder position is when the object is placed so that its base makes an angle with the H. P. but the position of the object with reference to the V. P. remains unchanged ; or the object is so turned that the face or the axis of the object which was parallel to the V. P. is turned to make an angle with the V. P. but the position of the object with the H. P. remains unchanged ; i. e. the object makes angle with one of the co-ordinate planes only.

The third or difficult case is when the object is in an oblique position with reference to the two planes of projection or when the object making an angle with the H. P. is turned so that its face or axis also makes an angle with the V. P.

The simple rules of finding the plan and elevation of an object when it is oblique or makes angles with both the planes of projection are the following :—

1. First place the object on one plane with one edge of the object parallel to the other plane and draw the plan or elevation first, which ever is easier, and draw the other view by projecting from the one already drawn and supplying the parts wanted. In the majority of cases it will be found easier to place the object on the horizontal plane.

2. Raise the object either resting on a corner or on an edge when that edge is at right angles to the ground line; the base making an angle with the H. P. but the axis of the object is turned parallel to the V. P. The base of an object in elevation is a straight line and it can be raised on one end when that end represents a corner or an edge of an object at right angles to the ground line. This will change the form of the plan but the elevation will not be altered in form but simply raised on one end of the base line at the same angle which the base makes with the H. P. Draw this figure on the right of the first elevation. As the distance of the points of the object from the V. P. is not altered the plan can easily be obtained by

drawing lines from the various points of the first plan parallel to the ground line and intersecting these lines by projectors from the corresponding points of the 2nd elevation. The second plan is formed by joining the points thus obtained.

3. Now retain the angle which the base of the object makes with the H. P. unaltered and turn the object so that its axis, which was parallel to the V. P. in the 2nd position, will make the required angle with the V. P. in the 3rd position. As the angle which the object makes with the H. P. is not altered, the 2nd plan will remain unchanged in form but only turned in position. Copy the 2nd plan on its right side placing the axis of the figure at the required angle with the V. P. The heights of the points of the object remain the same as they were in the 2nd position, therefore draw lines parallel to the ground line from the points of the 2nd elevation and intersect them by projectors from the corresponding points in the 3rd plan. The 3rd elevation is obtained by joining these points, and this and the 3rd plan complete the projections of the object in the particular position making angles with both the planes of projections. In the last figure the 3rd elevation presents to the eye a solid appearance as the object is so turned that by looking perpendicularly to the V. P. we see two faces and the base of the object i. e. the three dimensions of solid.

PROBLEM. 1.

Project a cube with edges 1½ inches long resting on one edge on the H. P. and its base makes an angle of 30° with the H. P. Fig. 237.*

ABCD is the plan of the cube half full size and numbered 1. It is drawn with one side AD perpendicular to the ground line XY as it is to be raised on one edge. The elevation of the cube is obtained by projecting from the points ABCD in plan and measuring the height on the projectors from the ground line. It is shown in No. 2. Its bottom corners are named in small letters corresponding to those of the plan and the top corners by the same letters with a dash. The elevation *abcd* is raised at an angle of 30° with X Y and resting on the edge *ad* in No. 3. The last plan or No. 4 is obtained by projecting vertically down the corners of No. 3 and getting the projectors intersected by projecting horizontally from the corresponding corners of No. 1. As No. 3 is obtained from No. 2 by turning it on one corner parallel to the V. P., the distances of the points ABCD top and bottom from the V. P. remain fixed in its altered position No. 3; they are determined by drawing lines parallel to XY from their first position in No. 1. It is drawn by the first two rules of projection.

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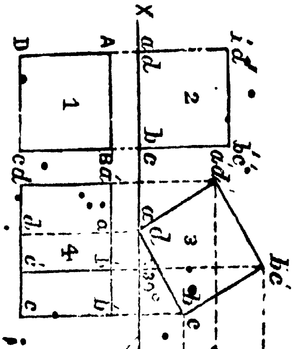
* Numbers of figures in Part II continue from Part I.

2. Project a cube edges 1¼" long resting on one edge on the H. P., its base making an angle of 30° with the H. P. and two of its faces in vertical planes inclined to the V. P. at 45°. Fig. 238.

This is a case of an object making compound angles with the two planes of projection and is to be drawn by the three rules of



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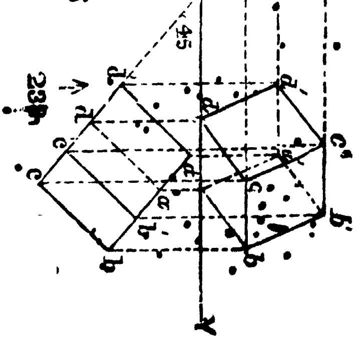

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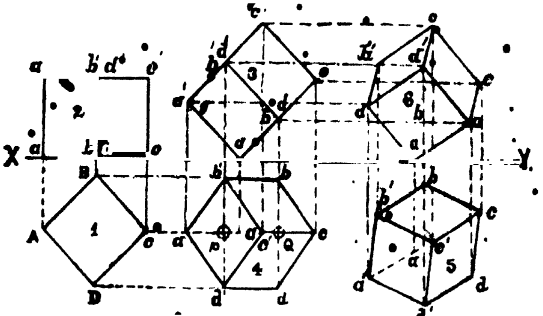
projection. The two projections of fig. 237 solve the first part of this problem and No. 4 fig. 237 is the plan of the cube with its base raised at an angle of 30° with the H.P. In fig. 238 below XY place No. 4 of fig. 237 with two faces at 45° with XY, that is the lines d'dc'c and a'ab'b representing the front and back faces of the cube which were parallel to XY are turned 45° from it. This is the new plan in its altered position. The elevation is obtained by projecting upwards from the corners of the new plan and measuring the hights of the corresponding points above XY from No. 3 fig. 237. Here the three faces of the cube are seen. There need be no difficulty to the beginners in finding out the firm and the dotted lines of the figure. The plane d'dc'c is first seen in the elevanion as shown by the arrow head. So dc and cc' are firm lines and no firm line can appear through d' d c' c as it is in front, therefore d'a', a' b' and a' a are dotted lines.

3 Project a cube with edges 1 inch long resting on one corner on the H P. its base making an angle of 45° with H P and the plan of its axis is turned at an angle of 30° with the V. P. Fig. 239.

As the cube rests on one corner it is to be placed with the diagonal AC of the base parallel to XY. No 2 is obtained by projecting from No. 1 and measuring the heights. No. 3 is a copy of No 2 with the base line *abcd* raised at an angle of 45° with the H P. No. 4 is obtained by projecting from No. 3 and No. 1. In No. 4. the axis PQ is parallel to the V.P. *i.e.* the ground line in the horizontal plane. No. 5. is a copy of No. 4 with its axis PQ at an angle of 30° with XY. The last elevation No. 6 is obtained by projecting vertically upwards from the points of No. 5 and inersecting them by horizontal projectors from the corresponding points of No. 3. The correctness of No. 4 rests on the accurate copy of No. 2 in its altered position No. 3 and that of No 5 depends on the correct copy of No 4 in the altered position, No. 5.

4. Project a cube with an edge in front and two faces at 45° with the V. P. and resting on one corner in such a position that a solid diagonal is vertical. Fig. 240.

This is not a case of an object making compound angles with the two planes of projection. The two faces of the cube make 45° with V.P. is simply to place the cube as in No. 1 fig. 239. A solid diogonal means the straight distance through the solid from a corner of the bottom to the opposite corner of the top as AC,



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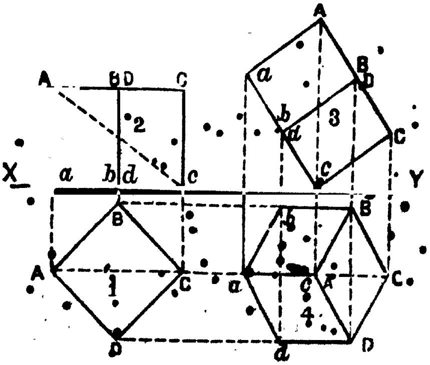

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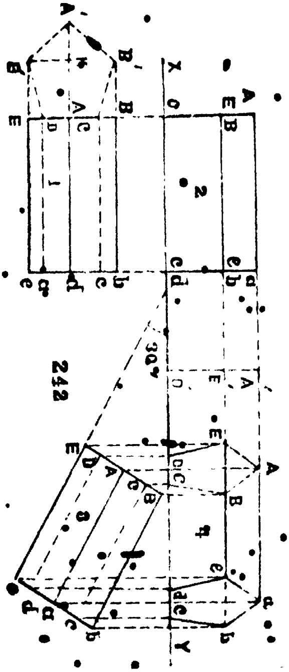
A is a corner of the top plane on the left and c is the opposite corner of the bottom on the right Here the solid diagonal is to be taken parallel to the V.P. and is shown by its full length on the elevation. No. 3 is a true copy of No. 2, with the diagonal Ac, vertical. No. 4 is the view required obtained by projections from No. 3 and No. 1. This is a particular case of projection in which all the edges of the cube are equally inclined to the ground, as all the edges of the cube in the plan No. 4 appear equal. The particular projection is called the isometric projection to be specially treated in Chapter III.

5. Project a triangular prism resting on one edge of the triangular end which is equilateral of 3/4″ side. The lower face makes 30° with the H. P and the longer edges 2″ in vertical planes inclined at 45° with the V. P. Fig. 241.

This is a case of compound angle which will require 3 sets of projections. As the prism will rest on an edge of one end that edge is to be placed so as to appear as a point in the elevation, *i.e.* it is to be placed at right angles to XY The figure is drawn half full size. Imagine the end towards BC is placed on the ground, that is A′ BC is an equilateral triangle. By projecting from A′ the position of the ridge or top edge AD is obtained. The height in No 2 is the altitude of the equilateral triangle A′ BC *i.e.* A′A. In No 5 the end DEF and the base BCFE are in front so we see them in elevation, as the edge AB passes over the plane BCFE in elevation it is dotted. The edges AC and CB are seen, not for the plane ABC which we cannot see but for the other two faces ACFD and BCFE which we see. The line AB in fig. 6 is a dotted line.

6. Project a regular pentagonal prism 2″ long with sides of base 3/4″ lying on one face on the ground with the plan of the longer sides inclined at 30° with the V.P. Fig. 242.

The fig. is drawn half size. Take a straight line DC at right acngles to XY and equal to one side of the pentagon. On DC Aonstruct a regular pentagon (Fig. 80 ch. VI part I) as A′ B′ CE′ D′. Produce DC both ways. Imagine the pentagon A′ B′ CDE′ to be vertically on DC and get the corners B′ and E′ projected on DC produced as B and E. Then complete the plan and elevation Nos. 1 and 2. In elevation the height CA = AA′. The side DE′ will appear in elevation as EC = AK. Place No. 1 as No. 3 with longer sides at 30° with XY and obtain No. 4 by projecting from No. 3 and No. 2.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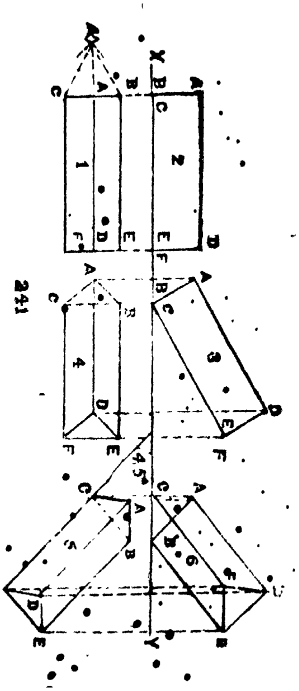
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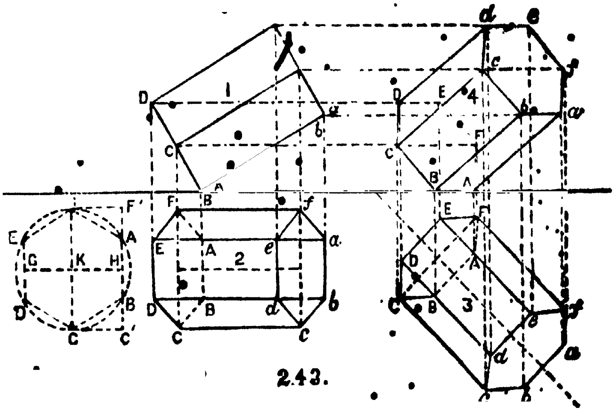
NOTE :—No. 4 can be drawn by one projection as No. 1 can be placed as No. 3 directly by drawing the first straight line dc at 60° with XY and completing the pentagon on it. The heights of the elevation can be projected from the line A'E'D'=A'KA of the pentagon.

7. Project a regular hexagonal prism 2¼" long with sides of base 1" resting on one edge of base, the base raised at an angle of 30° with the ground and the axis, in a vertical plane inclined at 45° to the V.P. Fig. 243.

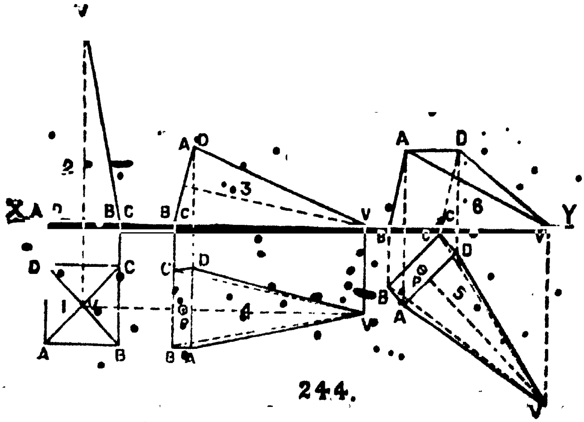
The problem can be drawn by two sets of projection instead of three as shown in the figure. The preliminary set that has been omitted is simply to obtain the angular points in plan and elevation which can be found by the drawing of a hexagon ABCDEF and projecting points on AB produced as F'ABC' and by taking a line perpendicular to BA and projecting the corners on that line as GKH. Draw No. 1, the elevation first, as it is the simplest figure, by taking a line at an angle of 30° with XY, representing the base BbaA. Draw BCD at right angles to Bb and equal to GKH and complete No. 1. Project from the points of No. 1. and from the 4 points F'ABC' and complete No. 2. Place No. 2 with the axis line PQ at 45° with XY and complete No. 3. No. 4 is obtained by projecting from No. 3 and No. 1.

8. Draw plan and elevation of a square pyramid 2¼" high and 1" edge of base with one of its triangular faces on the horizontal plane and its axis in a vertical plane inclined at 45° with the V.P. Fig. 244.

As the pyramid lies on the ground on one of its triangular faces, its base is at an angle with the ground and as its axis is inclined to the V. P. it is a case of compound angle. It is to be drawn by 3 sets of projections. Draw a square ABCD with the diagonals AC and BD intersecting in V, the vertex, No. 1, as the plan of the pyramid resting on its base on the ground, with the edge BC of the base, on which it is to be turned for placing the pyramid on the triangular face BCV on the ground, at right angles to XY. No. 2 is its elevation obtained from No. 1 and the height of the pyramid. No. 3 is a copy of No. 2, with the line BV' on XY. Obtain No. 4 by projecting from No. 3, and No. 1. Place No. 4 with the axis line PV at 45° with XY as No. 5. No. 6 is obtained by projecting from No. 5 and No. 3. The face ABV is in front so it appears the biggest in the elevation. The base ABCD and the face DCV are away from the observer and the line CD is a dotted line.



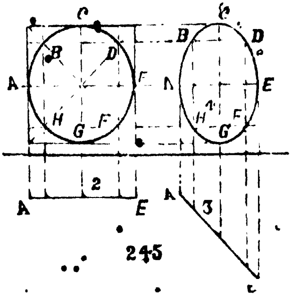
2.43.



244.

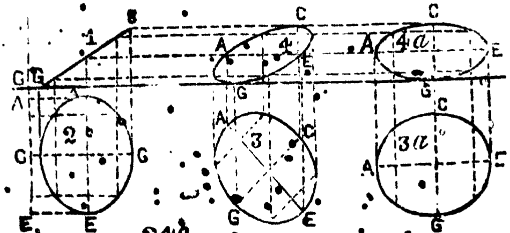
9. Draw plan and elevation of a circle of 1½" diameter. (1) With its plane vertical and parallel to the V. P. (2) with its plane vertical and inclined to the V.P. at 45°. Fig. 245.

The circle is drawn first in elevation as it is parallel to the V. P Its plan is a straight line. All projections of circles are drawn by describing a square about it and obtaining 8 points in its circumference as ABCDEFGH ; the diagonals of the square intersect the circumference in 4 points and the other 4 points are where the sides of the square touch the circumference, dividing it into 8 equal parts. No. 3 is a copy of No. 2 at an angle of 45° with XY. No. 4 is obtained by projecting from No. 3 and No. 1.



245

10. Find the plan and elevation of the circle when the plane is inclined to the ground at 30° and to the V.P at 45°. Fig. 246.



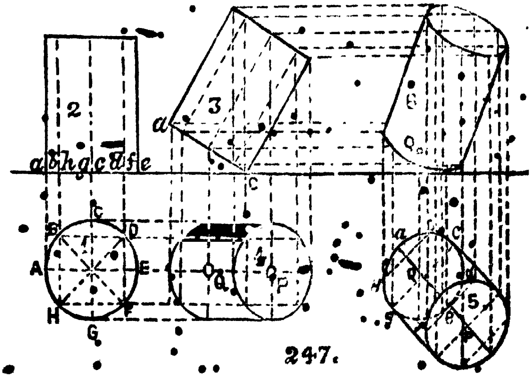
246.

In No. 4 fig. 245, the circle is inclined to the V. P. at 45° and to turn it at 30° with the ground it is to be turned on the diameter CG in a direction at right angles to AE No. 3, fig. 245, which requires the knowledge of descriptive geometry, a higher branch of projection. There is a simpler way of solving the problem. Place the circle first at right angles to the two planes of projection as GC and AE in fig. 246. The divisions in GC and AE are taken from No. 1, and No. 2 fig. 245. Incline GC at 30° to XY that is the plane of the circle is first inclined to the ground at 30° and number it as 1. Project the plan No. 2 from No. 1 and the points in AE. In No. 2 dia AE is at right angles to the ground line i.e. to the V. P. Place No. 2 as No. 3 with dia. AE at 45° with XY (compare with AE No. 3 fig. 245). Complete the elevation No. 4 by projecting from No. 3 and No. 1.

11. Draw plan and elevation of a circular plane of 1½" dia. when its plane is inclined to the ground at 30° and to the V.P. at 60°. Fig. 246.

This is possible when the diameter AE is turned 90° from its original position No. 2. in fig. 246. i.e. from the right angle to parallel to XY. No. 2 fig. 246 is copied as No. 3a fig. 246. From No. 3a and No. 4 complete No. 4a. The case will be more clear when we deal with lines in the next chapter.

12. Project a cylinder, diameter of base 1½" and height of axis 1½" resting on its circular rim, the base raised at an angle of 30° to the ground and its axis is in a vertical plane which makes 45° with the V.P. Fig 247.

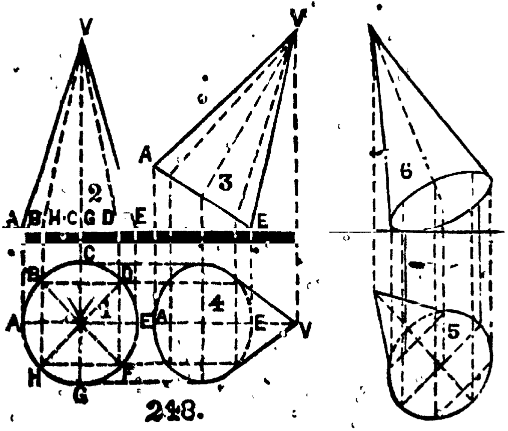


247.

Place the cylinder first vertically and draw No. 1 and No. 2

Plan and elevation of the cylinder. Obtain ABCDEFGH the eight Points in the circumference and project them on the top and bottom of the cylinder in No. 2 Place No. 2 as No. 3 in elevation with the base *ac* at 30° with XY. and project No. 4 from No. 3 and No. 1. Place No. 4 as No. 5 with the axis line PQ at 45° with XY and project No. 6 from No. 5 and No. 3. It will be seen that in elevation No. 6 the points in the lower curve are not exactly those which are seen in the plan No. 5 as the points f, e and d appear the points a, b, and c disappear. Notice that in all the cases Nos. 2, 4 and 6, the cylinder retains its full breadth, as whatever be the position, one diameter of section appears perpendicular to the sight. This is a check in the drawings of the beginners.

13. Project a cone, height 2¼ inches, diameter of base 1½", resting on its edge, with its base inclined at an angle of 30° with the H P. but its axis is in a vertical plane inclined at 45° with the V.P. Fig. 248.

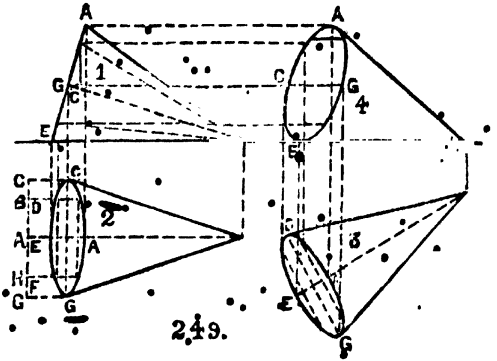


248.

First place the cone on its base on the H. P. and draw a circle in plan of diameter 1½" which is the plan of the cone. Number it as 1. Divide the circumference of this circle into 8 equal parts in ABCDEFGH for points for projection.

Obtain No. 2 from No. 1 and the height of the axis. Turn No. 2 on the point E of the base so that AE the diameter of the base parallel to the V. P. makes 30° with the H. P. in No. 3. Project No. 4 from the points in No. 3 and No. 1. In No. 4 the axis is parallel to the V. P. In No. 5 the plan No. 4 is copied with the axis at 45° with the V. P. Obtain No. 6 by projecting from No. 5 and No. 3. In No. 6 the axis being inclined to the V. P. appears less than its real length shown in No. 2 and No. 3.

14. Project a cone lying on its slant side on the H. P. with its axis in a vertical plane inclined at 30° with the V. P. Fig. 249.

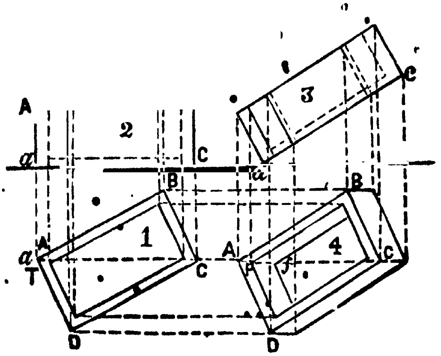


249.

In this case No. 2 of fig. 248 is placed with the slant side VE on XY and numbered 1. No. 2 is obtained by projecting from No. 1 and from the line CG taken from No. 1 fig. 248 and placing it at right angles to XY. No. 3 is a copy of No. 2 with the axis inclined to the V. P. at 30° and No. 4 is obtained by projecting from No. 3 and No. 1.

In No. 2 the horizontal diameter of base GC retains its full length which in No. 1 is a point as it is perpendicular to the V. P. and in No. 4 it is diminished as it makes 60° with the V. P.

15. Project a rectangular box without a lid resting on one corner with an edge in front the sides inclined to the V. P. and the base raised at 30° with the H.P. fig. 250.

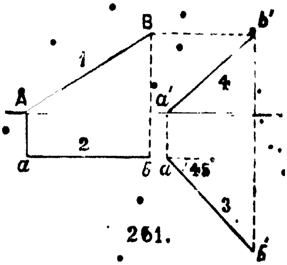


250.

Here the box is to be placed with the diagonal of the base parallel to XY, as the base is to be raised on one corner, *a*. Complete the rectangle ABCD on the diagonal AC placed parallel to XY. The capital letters are for the corners of the top and the small letters for the corresponding corners of the bottom. AC the diagonal of the top coincides with *ac* the diagonal of the bottom. Take AT as the thickness of planks and place it on the four sides. No. 2 is projected from No. 1 with the height of the box aA. The box is raised on the corner *a*, with the diagonal *ac* at 30° to XY and copy No. 2 in the position No. 3. No. 4 is projected from No. 3 and No. 1. To project the thickness of the box project the points of thickness of No. 1 on No. 2 and copy them on No. 3. Projecting from the points of the thickness of No. 3 and No. 1 the thickness of the box in No. 4 is obtained. The inside corner Ff of the box is seen in No. 4 and it is a firm line in fig. 250. A portion of the bottom of the box is seen from top.

A few cases of projections of lines and planes will further illustrate the rules of projection.

16. Draw the plan and elevation of a line 1" long inclined at 30° to the H.P. and the plan of the line is at 45° with the V.P. Fig. 251.

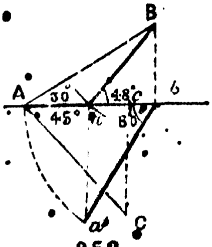


251.

Place AB 1" long at 30° with XY and project *ab* its plan parallel to XY. Place *ab* i.e No. 2 at 45° with XY as *ab* No. 3 and project *a'b'* No. 4 from No. 3 and No. 1. Here the plan of the line AB *i.e. ab* makes 45° with the vertical plane and not the line itself.

17. When the line is so placed as to make 45° with the V.P. at the same time when it is inclined at 30° with the H. P, the projection of it is explained in Fig. 252.

Place AB at 30° with XY fig. 252 and draw AC equal to AB at 45° with XY. Project the points B and C on XY and obtain Ab and Ac which represent the lengths of plan and elevation respectively of AB the given line. These two lengths are to be placed with reference to the ground line so that one can be projected from the other. As the angle with the H. P. remains unaltered when the line moves from its first parallel position with the V. P. to its angular position of 45°, it can only be turned by keeping one of its ends fixed in position with relation to the V.P. Assume the end B to remain fixed.



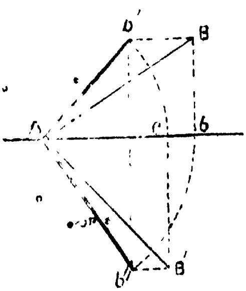
252.

With centre B and radius Ac (the length of the elevation), draw an arc, cutting XY in *a*. Join Ba then Ba is the projection of the elevation. With b as centre and bA (length of plan) as radius, draw an arc on which the moving end A of the line lies. To find out this point project from a and find a' where it intersects the arc. Join a'b which is the position of the plan of the line. It will be noticed that the length of the plan is the same as that in fig. 251

as the angle with the H.P. remains unaltered but the length Ba of the elevation fig. 252 is less than a'b' the elevation in fig. 251 as the angle which the plan in fig 252 makes with XY is greater than 45°, the angle of the plan in 251. The angle Aba is nearly 60°. When the plan makes 45° with XY the angle which the line really makes with V. P. is much less. It may be noticed here that angles increase in plan and elevation as the projections of the lines containing them must decrease in plan and evevation. This is a case of solid geometry.

18. Draw the projections of a line 1¼ inches long which has one end in XY and is inclined at 30° to H.P. and 45° to V.P. Fig. 253.

It is the same case as fig. 252 with the difference that the end which turns from the V. P. is not A but B the upper end of the line. The end A of the line is on XY which remains fixed Ab and Ac are the lengths of the plan and elevation respectively, they are to be placed in their proper positions. As the line is turned on the point A with the angle it makes with H.P. unaltered, the point B turns with its height from the H. P. remaining the same. Draw a horizontal line from B, and with A as centre and Ac as radius draw an arc cutting the horizontal line from B at b'. Join Ab' which is the elevation. With A as centre and Ab as radius draw an arc and project b" on it from b'. Join Ab" which is the plan.

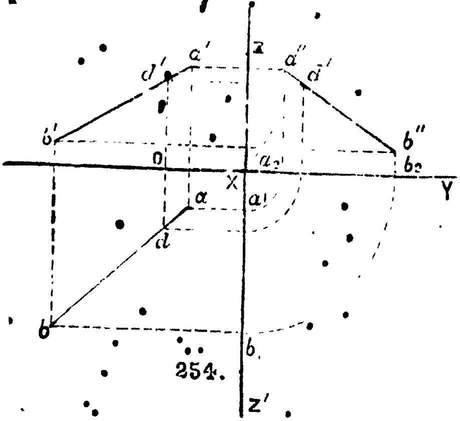


253.

19. A line is given by its plan and elevation; find the shortest distance of the line from XY. Fig. 254.

Let $a'b'$ and ab be the elevation and plan respectsvely of the line AB. It is required to find the length of the perpendicular that falls on the line from a point in the ground line. The perpendicular will be seen fully on a plane at right angles to both the H. P. and V. P. that is on the side view. Draw ZXZ' at right angles to XY then ZXY is the side plane. Project the point b_1 and a_1 on XZ' from b and a of the plan. With

X as centre transfer the points b_1 and a_1 as b_2 and a_2 on the ground line. By projecting from b_2 and a_2 and from b′ and a′ of the elevation the side elevation b″a″ is obtained. From X



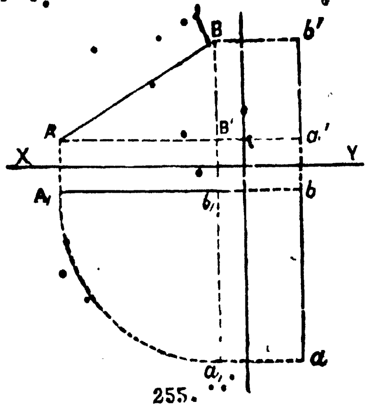
254.

where ZZ′ intersect the ground line draw xd″ perpendicular to b″a″; then xd″ is the shortest distant of the line AB from XY. To represent xd″ on the front view project from d″ on a′b′ in elevation as d′ and from d′ project d on the plan ab. Then od and od′ are the plan and elevation of the shortest line.

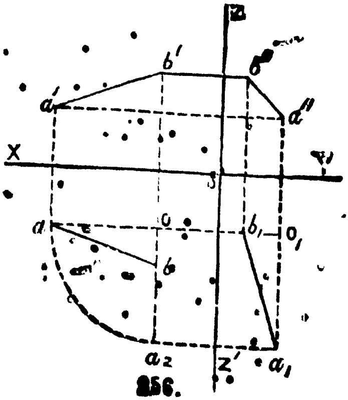
20. Draw the projections of a line 1¼″ long inclined at 30° with the H. P. and 60° with the V. P. Fig. 255.

Place AB at 30° with the horizontal plane. AB′ is the length of the plan. As the line AB is at 30° with the H. P. it can not incline more than 60° with the V. P. which will be the case when it revolves 90° on B as centre. Imagine the first view drawn as the side view of the figure, then AB is seen full length, the angle at A is 30° and at B is 60°. Draw a line parallel to XY and project on it the points A and B as A_1 b_1. With b_1 as centre and b_1 A_1 as radius draw a quadrant and obtain b_1 a_1 parallel to ZZ′ the edge of the front plane. Draw ab parallel to a_1b_1 and a′b′ parallel to B′B and equal to them respectively. Then ab is the plan, and a′b′ the elevation of the line AB.

21. Two points are respectively ½" and 1¼" from both planes of projection and their distance on the horizontal plane is 1½". Find the projection of the line joining the two points on a plane at right angles to the two planes of projection. Fig. 256.



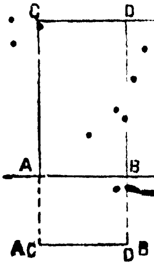
255.



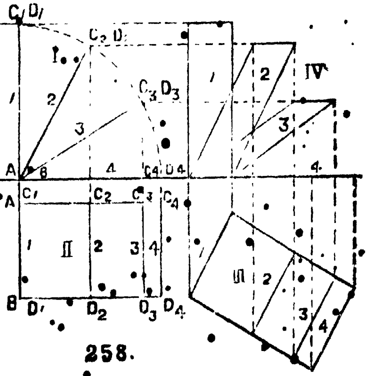
256.

Draw a a′ and b b′ the plan and elevation respectively of the two points A and B. Scale ½. The distance ab is 1½ inches. ab is the plan and a′b′ the elevation of the line AB. With O as centre and Oa as radius draw a quadrant aa_1. Project the point O on a line parallel to ZZ^1 and mark it as b_1; measure b_1o_1 parallel to XY and equal to *bo* and from o_1 draw a line parallel to ZZ^1 and project on it the point a_1 from a_1. b_1a_1 is the plan on a plane at right angles to the two planes of projection. b″a″, the side elevation is obtained by projecting from the plan b_1a_1 and the first elevation b′a′.

22. Draw the projections of a rectangular plane when it rests on its shorter edge on the ground, its long edges are at a particular angle with the V. P and the plane is gradually inclined at various angles to the H.P. Fig. 258.



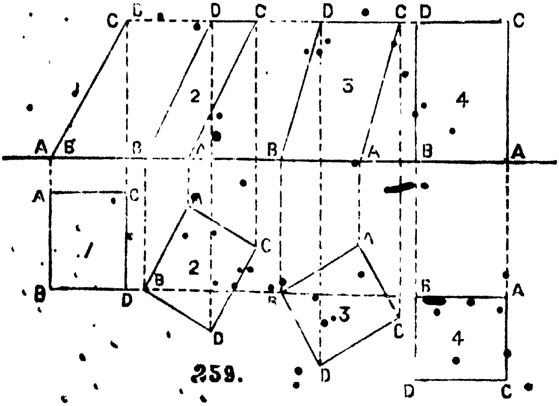
257.



258.

The rectangular plane is shown in Fig. 257 as ABCD a rectangle in elevation and AC BD a straight line in plan. Place the rectangle at right angles to both the planes of projection as No. 1 fig. 258. Incline No. 1 in the elevation as 2, 3 and 4 and obtain the three plans when the plane is inclined at 60°, 30° and 0° with the H. P. The elevation is numbered as I and plan as II. Place the whole of the plan II in the 4 positions as III of fig. 258 with the long sides at a particular angle to the V. P. and project the elevation IV fig. 258 from III and I. For the convenience of showing the different angles with the H P. the plane is first placed at right angles to the V. P. and is gradually turned to H. P. It will be noticed that the horizontal widths of the elevations remain unaltered as the widths being parallel to the H. P. retain the same angle with the V. P., but the inclinations of the long sides with the V. P. which though coincide in the plan change as the angle they make with the H P. changes. This is the cause of the differences in the long sides of the figures in the elevation No. IV. This is a case of solid geometry and can be understood by a little thinking.

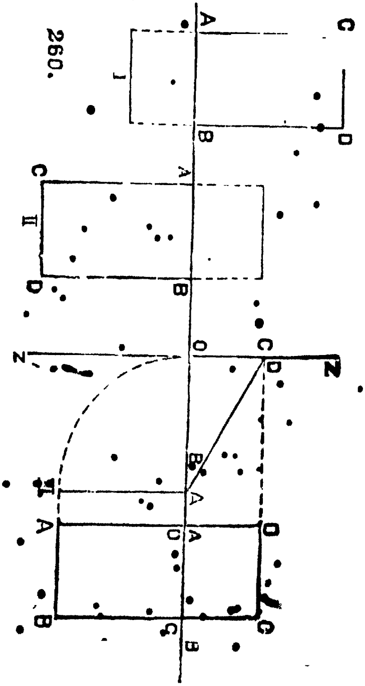
23.. Draw the projections of a rectangular plane when it rests on its shorter edge on the ground, its plane is inclined at an angle to the H. P. but the long edges are gradually turned at various angles from the V. P. Fig. 259.



259.

Place the rectangle ABCD of fig. 257, inclined to the H.P. at 60° and obtain the plan and elevation which are the same as No. 2 of I and II of fig. 258 and number it as 1. Copy plan No 1 as plan No. 2 with edges AC and BD inclined to the V.P. at 30° and project the elevation No. 2 from plan No. [illegible] and elevation No. 1. Place plan No. 1 as plan No. 3 with AC and BD at 60° with the V.P. and project the elevation No. 3 from plan No. 3 and elevation No. 1. Place plan No. 1 as plan No. 4 with AC and BD at 90° with the V.P. and project the elevation No. 4 from plan No. 4 and elevation No. 1. The heights of the planes in the elevations remain unaltered as the angle which it makes with H.P. is unchanged but the width AB increases as its angle with the V.P. decreases.

24. Draw the projections of a rectangular plane when its inclinations to both the planes of projection are together equal to 90 fig. 260 I, II, III.



260.

There are three positions in which the rectangular plane makes 90° with the two Co-ordinate planes.

I. When it is perpendicular to the H.P. but parallel to the V.P.

II. When it is perpendicular to the V.P. but parallel to tl H.P.

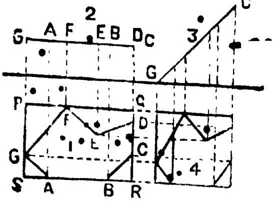
III. When one edge of the plane is on the H.P. and t opposite parallel edge is on the V.P. In this case the angl with the two Co-ordinate planes are complementary angles.

Case I. Plan is a straight line parallel to XY and the elev tion is the full rectangle ABDC.

Case II. Elevation is a straight line parallel to XY and tl plan is the full rectangle ABDC

Case III. The rectangle is placed with its plane parallel XY. Draw ZOZ' for the side plane The rectangular plane seen on the side view as a straight line inclined to the H.P. at a angle and to the V.P. at the complementary angle. The heigl in the elevation is OC and the length of the plan is OA. Fro these two dimensions and the width of the rectangle the pla ABCD and elevation ABCD is obtained as shown in case III fi 260.

Projection of irregular areas :—The projection c irregular areas can easily be found by inscribing them i rectangles, a side of which passes through a side of th irregular area and the remaining sides passing throug the corners of the figure Fig. 261.



261.

ABCDEFG is the irregular figure. The rectangle PQRS is described by drawing SR through AB and lines perpendicula to SR through DC and G. Draw PQ parallel to SR through I meeting the perpendiculars in P and Q

The plan ABCDEFG is raised at 45° with the H.P. on it corner G. No. 1 is the plan of the irregular plane with th corner G on the left. Its elevation is the straight line GC No. 2. No. 2 is placed at 45° with the H.P. and numbered as 3 No. 4 is obtained by projecting from No. 3 and No. 1.

CHAPTER II.

PROJECTIONS CONTINUED. SECTIONS AND DEVELOPMENTS.

There are four classes of projections :—

1. Orthographic, orthogonal or parallel projections, when the projectors are perpendicular to the planes of projection and are parallel to each other

Isometric projection is a particular case of orthographic projection.

2. Perspective, radial or natural projection, when the projectors converge to or pass through a point, the eye.

3. Oblique projection, when the projectors are parallel but inclined to the planes of projection. It is well fitted to show the forms and dimensions of what may be termed rectangular solids, as examples of wood-work.

4 Horizontal projection :—

In this projection only one plane the H.P. is used and figures are given indicating the heights of the points either above or below the H.P. No elevation is necessary. The figures are called the "Indices" and they are + or — as they are above or below the H.P. It is of great utility in the contour surveys and in the drawing of forts &c.

Definitions of solids :—

A *solid* is that which has length, breadth and thickness. The boundaries of a solid are surfaces either plane or curved.

The boundary lines of the surfaces of solids are called the *edges.*

A *solid angle* is made by the meeting of three or more plane angles, which are not in the same plane, in one point as the corner of a cube where the three angles meet.

A *prism* is a solid figure bounded by plane figures of which two that are opposite called *ends* are similar, equal and parallel to one another, and the others, called faces, are parallelograms ; in right prisms the faces, are rectangles.

A *pyramid* is a solid figure which has a plane figure for its base which may be a triangle or a polygon and each of its sides or faces are triangles whose vertices meet in a point above the base, called the apex of the pyramid.

Prisms and pyramids are named as triangular, square, pentagonal &c. according to the form of their bases.

A *cylinder* is a solid figure the two ends of which are circular planes connected by a curved surface. It is described by the revolution of a rectangle about one of its sides which remains fixed, and which is called the axis of the cylinder.

A *right cone* is a solid figure described by the revolution of a right-angled triangle about one of its sides containing the right angle and which remains fixed and forms the axis of the cone. The base of a cone is a plane circle the radius of which is the revolving side of the right angle

A *sphere* is a solid figure described by the revolution of a semicircle about its diameter which remains fixed.

When the upper portion of a cone or pyramid is cut off by a plane and removed the remainder of the solid is called the *frustum* of cone or pyramid and the solid is said to be *truncated*.

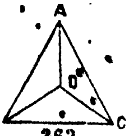
The solid formed by the revolution of a circular plane about a circumference passing through its centre is called a *ring*.

There are five regular solids and they are named after the number of faces they each possess *viz* :—

1. The *Tetrahedron* has four equal and equilateral triangles for its faces. It is a triangular pyramid. Fig. 262.

2. The *Hexahedron* has six squares for its faces. It is commonly called a cube. Fig 263.

3. The *Octahedron* has eight equal and equilateral triangles for its faces. It is a double pyramid on a square base. Fig 264.



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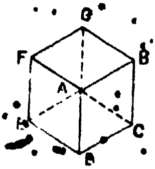

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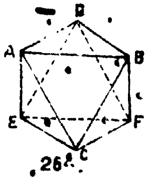
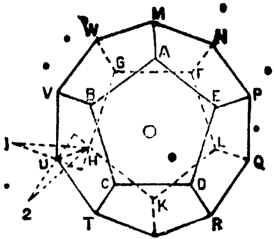


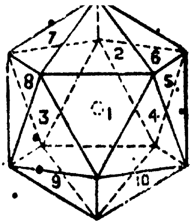
264.

4. The *Duodecahedron* has 12 equal and regular pentagons for its faces; the opposite faces are parallel to each other and their angles alternating. Fig 265.

5. The *Icosahedron* has 20 equal and equilateral triangles for its faces. The opposite faces are parallel and their angles alternating. Ten faces seen are numbered, ten dotted faces are below. Fig. 266.



265.



266.

These regular solids possess the following properties :— The faces of each solid are equal, similar in shape and its edges are of equal lengths.

These faces are all regular polygons.

If the solids were each inscribed in a sphere all their angular points would touch the concave surface of the sphere *i. e.* would be equidistant from its centre.

Projections of complicated or irregular solids may be conveniently solved by enclosing them in suitable parallelopipeds like irregular plane figures enclosed in rectangles. Fig. 261

Alteration of ground line :—

From the rules of projection already given, it is found that a plan or elevation is to be copied exactly in a different position to obtain a new elevation or plan. It is troublesome and is often a source of inaccuracy of the resulting figure. The final plan and elevation may be easily obtained by changing the position of the ground line to the angle the object makes with the particular plane and assuming fresh vertical or horizontal planes instead of changing the position of the plan or elevation. By this method more than one elevation can easily be shown from one plan. This is called auxiliary projection.

Rules for drawing auxiliary projections by the alteration of ground line are given below :—

An auxiliary elevation is the projection on any vertical plane not parallel to the principal vertical plane.

An auxiliary plan is the projection on any plane which is perpendicular to the vertical plane but not parallel to the H. P.

Rule I. To obtain the auxiliary elevation on a new ground line $X'Y'$ project from the plan perpendicular to $X'Y'$ and for the new elevation mark off on projectors the distances of the points above $X'Y'$ equal to the distances of the corresponding points of the old elevation above XY. The new ground line $X'Y'$ is drawn at the same angle with XY as the object makes with the V. P. for the new elevation.

Rule II. To obtain the auxiliary plan on a new ground line $X'Y'$ project from the elevation perpendicular to $X'Y'$ and for the new plan mark off on the projectors the distances of the points in front of $X'Y'$ equal to the distances of corresponding points of the old plan in front of XY. $X'Y'$ is drawn at the same angle with XY as the object makes with the H. P.

Sections and Developments:—

In many drawings the plan and elevation will not furnish all the information that is necessary to construct the objects. The details of the interior arrangements are required for the thicknesses of parts and for other portions which can not be seen from the outside. These are obtained by means of sections.

When an object is cut into two portions the representation of the surface that would be exposed, when the portion nearest to the eye is removed is called a *section* of the object: the plane cutting the object is imagined to pass in any required direction, in order the better, to show its internal form. It is convenient for ordinary projection to imagine the cutting plane either to be vertical or horizontal. There is a difference between the sectional plan and elevation and the true shape of section. The sectional plan and elevation are the views obtained by looking vertically down on the horizontal plane or perpendicularly to the vertical plane. The true shape of section is obtained only when the plane of projection is parallel to the cutting plane. The severed parts are indicated by section lines or series of inclined parallel lines.

The best position for a cutting plane is perpendicular to one plane and parallel to the other as it gives the true form of section from which measurements may be taken.

The *profile* of an object is a section made by a vertical plane cutting the object in a direction perpendicular to its length. A profile shows the true width of the different parts. Useful in earth work.

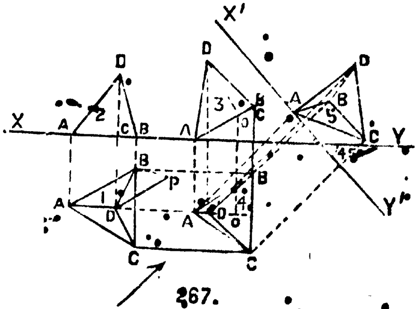
A *Contour* is the plan of the intersection of a surface by a horizontal plane, useful for delineating undulating or sloping grounds.

Development:—

The surfaces of all solids with plane faces and also certain curved surfaces, including the cone and cylinder can be *developed*, that is unfolded into one plane without any wrinkling or stretching. This is useful in making objects of sheet metal as the shape of the development is to be first cut out from the sheet to prevent wastage.

Five examples of harder projection to be drawn by changing the position of the ground line, *i.e.* by the rules of Alteration of ground lines.

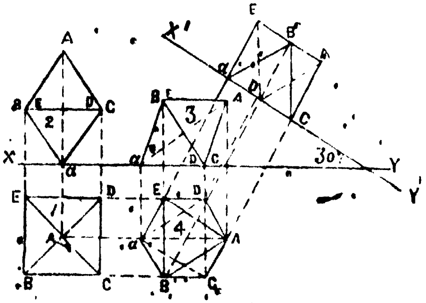
25. Draw plan and elevation of a tetrahedron of 2" edge resting on one corner, the base raised at an angle of 30° with the H. P. and the axis in a vertical plane at 45° with the V. P. Fig. 267.



267.

As the tetrahedron is to be raised in one corner place the equilateral triangle of the base ABC so that one edge BC will be at right angles to XY then the solid can be raised on the corner A of the base. This is No. 1. Project elevation No. 2 from plan No. 1. The height of the apex D is obtained by drawing DP perpendicular to CD and from C as centre and with CB one edge of the solid as radius draw an arc BP cutting DP in P. Then DP *is the height of the point D from the base. Project from D and make CD in elevation equal to DP.* Join DA, DB. Then DAB is the elevation No. 2 and the line AB represents the base ABC which is to be raised on A at 30° to the ground. Draw AB in No. 3 at 30° with XY and copy No. 2 on AB. Project No. 4 from No. 3 and No. 1. The next elevation can be drawn without copying the plan No. 4 with the line DO (the plan of the axis) at 45° with XY. Instead of it draw a new ground line X'Y' at 45° with XY, and project from No. 4 on this new ground line. The heights of the points above X'Y' of the corners of the tetrahedron can be measured from the points of No. 3 above XY. The arrow head shows how No. 5 is viewed. Only the face ADC is seen as it is in front and obstructs the view of the other faces. The lines AB, BC in No. 5 are dotted lines

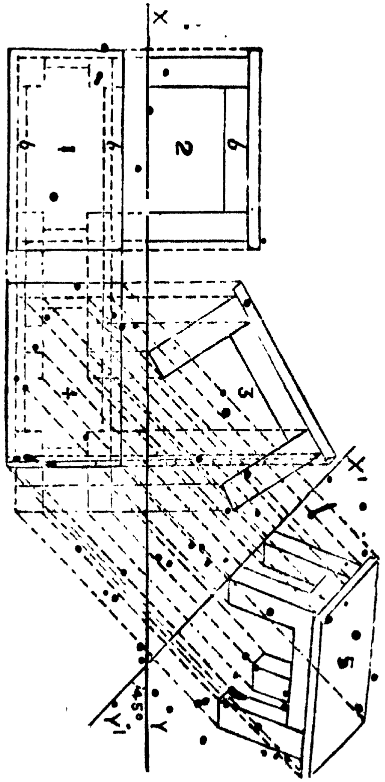
26. Project an octahedron with edges one inch long resting on one face on the H. P. The edge nearest to the V. P. makes 30 with it. Fig. 268.



268.

The octahedron is placed in plan No. 1 as a square pyramid in such a way that its two triangular faces ADC and ABE may appear as two straight lines AC and AB respectively in elevation No. 2. The height a'A of the octahedron is equal to a diagonal EC of the common base. The height a A is bisected and BC is drawn through the point of bisection. No. 3 is No. 2 thrown on face aDC on XY. No. 4 is projected from No. 3 and No. 1. The line ED one edge of the solid is nearest to the V. P. and parallel to it. This edge is to be turned at 30° with XY. Take a new ground line X'Y' at 30° with XY and project on it from No. 4. The heights of the points in No. 5 is obtained from the heights of the points in No. 3 above XY.

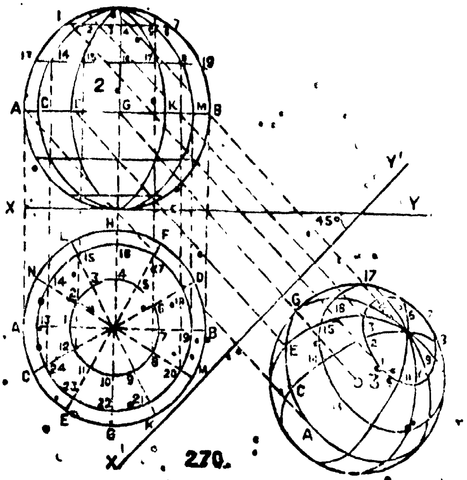
27. Project a table with 4 square legs resting on two legs and the other two legs are so raised that the top inclines to the H.P. at 30°. The table is then turned till the longer edges are in vertical planes which make 45° with the V. P. Fig. 269.



269.

Draw first the plan of the table with the legs as dotted lines No. 1. From No. 1, project No. 2, and put the height of the legs and the thickness of the top plank. The batten piece b is also seen. No. 2 is placed as No. 3 resting on two legs and the other two legs so raised as to make the top plank incline at 30° with the ground. Project plan No. 4. Then take a new ground line X'Y' at 45° with XY and project from No. 4 on this new ground line. As many lines, if drawn all together, confuse one another, it is advisable to project the top plank first and draw it by taking the heights of the points from No. 3 above XY. Then project one leg and draw on gradually. The required elevation is No. 5.

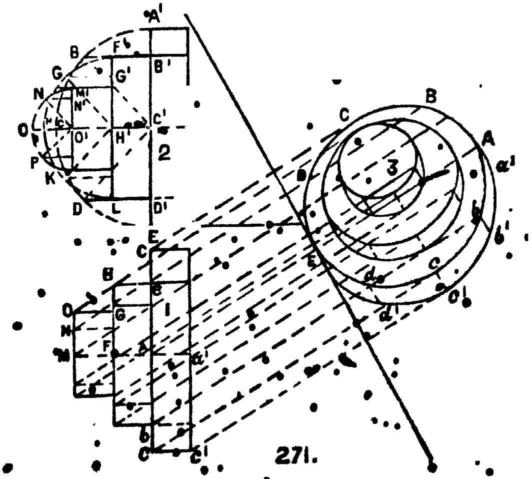
28. Project a sphere 2″ diameter with meridians and latitudes and with its axis inclined to the H.P. at an angle of 45° but parallel to the V.P. Fig. 270.



270.

Draw two circles in plan and elevation No. 1 and No. 2 representing the sphere and divide the circumferences of each into 12 equal parts for the plan and elevation of the latitudes and longitudes at distances of 30°. The latitudes appear in plan No. 1 as circles and in elevation No. 2 as straight lines parallel to XY. The longitudes appear in plan No. 1 as radial straight lines and in elevation No. 2 as ellipses. Number a few of the points in plan and put the same numbers on the corresponding points in elevation. Take a new ground line X'Y' at 45° with XY and project on the new ground line from the elevation No. 2, as plan is required to be drawn. Take distances of the point from X'Y' in plan No. 3 equal to the distances of the corresponding points in plan No. 1 from XY. First finish the latitudes. To avoid drawing dotted lines draw the great circle of the sphere in plan No. 3 by projecting the centre of elevation No. 2 on X'Y' and measuring the distance of the centre of No. 1 from XY. Where the latitudes meet this outer circle they can be drawn up to those points. Mark the pole, the upper one. Join the points of latitudes passing through this pole for the longitudes, ending the curves on the outer circle.

29. Draw plan and elevation of a set of three speed pulleys 2½″, 1½″ and 1″ diameters, respectively, and thickness ½ an inch each, resting on the rim of the larger pulley, and their faces making angles of 30° with the V.P. Fig. 271.

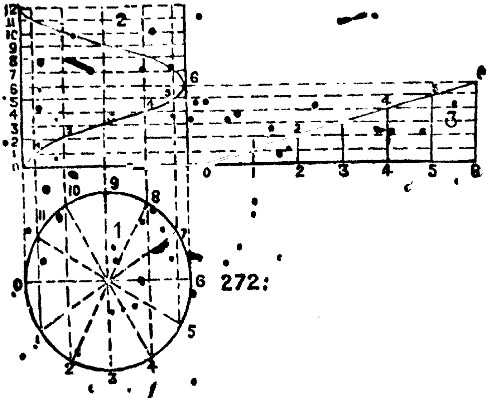


271.

Draw plan and elevation of the speed pulleys, resting on the rim of the largest pulley and as seen from one side. (No. 1 and

No. 2). The middle points M, F and A of the faces of the pulleys in plan No. 1 are the top points in the elevation No. 2 which are similarly lettered. The middle points O', R' and C' of the elevation No. 2 are the top points O, H and C of plan No. 1. Draw the semicircles A'BCDE on A'E the 1st pulley, the semicircle F'GHKL on F'L, the 2nd pulley and the semicircle M'NOPQ on M'Q the 3rd pulley for finding the points of projection on the rims. From the centre points of the faces draw lines at 45° upwards and downwards in the 3 semicircles to cut the arcs. Project these points of the arcs on the faces and the points B',C',D' &c. are obtained Draw a new ground line X'Y' at 60° with XY then it will be at 30° with the faces of the pulley. From the points in plan No. 1 project on the new ground line, and the heights are taken from the elevation No. 2 The heights of the points ABCDE in elevation No. 3 are the same as A'B'C'D'E' in elevation No 2. The thickness CC' in No. 3 of the pulley is obtained by projecting C' in plan No. 1 on elevation No. 3, and setting this measurement CC' on lines drawn parallel to X'Y' from the points A, B, C, D and E of elevation No. 3. Thicknesses of the other two pulleys are similarly obtained. It is necessary to project only one thickness

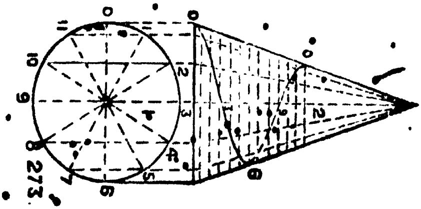
30. Project a helix of a given pitch on a vertical cylinder. Fig. 272.



272

The pitch is the distance between two turns of a helix measured parallel to the axis of the cylinder. Take the pitch of the helix to be one inch and the diameter of the cylinder, 1⅜″. Draw a circle of 1⅜″ dia showing the plan of cylinder, No. 1. Divide the circumference of circle No 1, into 12 equal parts for the 12 equidistant points on the helical curve. Project the elevation of the cylinder, No 2 and take spaces of 1″ on the side of it. Divide one space into 12 equal parts and number them from 0 to 12. Draw lines parallel to the ground line from these points. Project from the points 0 to 11 in plan No. 1 and where the projections from the points in plan, intersect the corresponding parallel in the elevation, are the points in the curve in elevation No. 2. Join these points. The curve from 0 to 6 is in front and so drawn firm and from 6 to 12 the curve is on the back of the cylinder which is drawn dotted. The 3rd figure is the development of half of the helical curve. Take the distances 01, 12, 23, 34, &c of the circumference in No. 1 on the ground line as 01, 12, 23, 34, &c and draw perpendiculars from these points where these intersect the corresponding horizontal lines from elevation No. 2, are the points of the developed curve which is a straight line in this case.

31. Project the curve of a helix of a given pitch on a given vertical cone. Fig. 273.

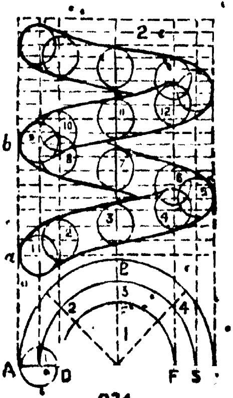


273.

Draw a circle representing the plan of the cone, No 1 and divide the circumference into 12 parts. Project the elevation, No. 2 from plan No 1. Suppose there are two turns of the helix on the whole height. Bisect the height and divide the lower half into 12 equal parts. Join the points of the circumference in plan No 1 with the centre. These radial lines are the lines on the surface of the cone from the vertex to the points in the base. Project these lines on the elevation, No 2 where these lines meet the corresponding horizontal lines, are the points of the curve in elevation No 2. Join these points for the curve.

The development of this curve will be shown in the portion dealing with the subject.

32. Project a thick 'wire' wound up into a helical curve or spring. Fig. 274.



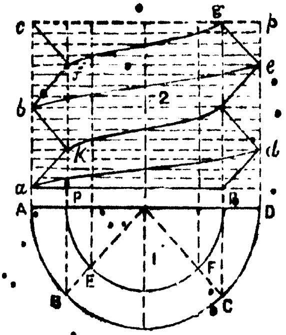
274.

This case differs from Prop. 30 fig. 272 in having a thickness for the curve turning up a cylinder instead of a thin line. The thickness is represented by two parallel curves touching the circumferences of small circles representing the circular section of the thick wire.

In plan draw AC the diameter of the cylinder which envelops the helix. Take AD and FC on the two ends representing the thickness of the wire or the diameter of its section. Then DF is the diameter of the hole of the helix.

Draw ABC and DEF semicircles for half plan. Bisect AD and FC at 1 and 5 respectively and draw a semicircle on 15 representing the half plan of the locus of the centre points of the sections of the wire. Let *ab* be the pitch of the helix in elevation No 1. Divide *ab* into 8 equal parts *i.e.* as many parts as the circumference of the plan is divided. As half plan is drawn only 4 divisions are shown. Mark the centre points in the plan as 1, 2, 3 &c and project from these centre points to intersect the horizontal lines from the corresponding points in elevation. Points 6, 7, 8, coincide with the projections from the points 4, 3, and 2 respectively. Draw circles from these points in elevation of diameter equal to the thickness of the wire and join the circumferences of these circles by tangential curves as shown in figure 274.

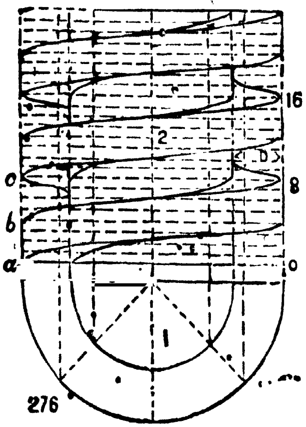
33. Project a V threaded screw ¼" pitch and ½ inch in depth, cylinder 2½ in. diameter. Fig. 275.



275.

Cylinder is the rod from which the screw is cut and its diameter is the diameter of the cylinder. Draw two concentric semi circles ABCD and PEFR representing the half plan of the screw. AP and RD are the depths of the thread on the two sides. Project the elevation and take *ab*, *bc* to be two consecutive pitch of the screw. Divide the semi-circumferences of the plan into 4 equal parts. Divide one pitch in elevation, No. 2 into 8 equal parts and draw from these points lines parallel to XY, and projecting from the points on the outer circumference of the plan, find points in the elevation of the outer helical curve and join these points. The curves *ad*, *be* in elevation representing the edges of the V thread are thus obtained. Join the points *a*, *d*, *b*, *e* and *c* with the points in the projections from P and R, 4 divisions up and down for the sides of the V threads. Now the curve of the root of the thread is to be drawn. The inner helical curve is commenced from K the 4th point in the projection from P and ends in the 8th point in the projection from R. The other points are found by projecting from the points in the inner curve of the plan.

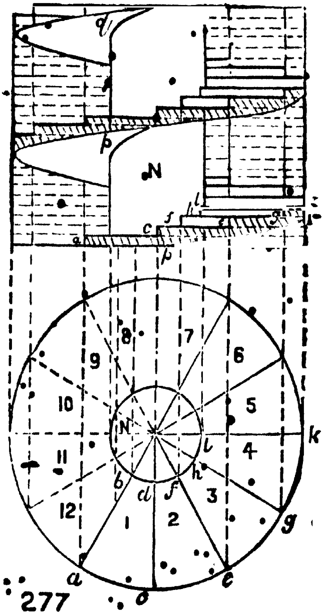
34. Project a square threaded screw depth ½ inch and pitch 1 inch. Cylinder 2½ in. diameter. Fig. 276.



276

Square threaded screws have the same thickness of thread from the root to the outside and so two parallel helical curves are to be drawn showing the thickness of the thread at the edge. Part of the inner cylinder is seen as the spaces are of the same thickness from the edge to the root. The pitch *ac* in elevation is bisected in *b*, *ab* is the thickness of the thread and *bc* is the space. The outer helical curves are first drawn from the points in the outer semi-circle of the half plan No. 1. The inner curves in elevation No 2 are commenced from the points in the horizontal lines Nos. 0 8 16 &c. of the elevation where the projections from the ends of the diameter of the inner curve in No. 1 meet.

35. Draw the elevation of a round staircase, the enclosure wall removed, showing the steps and Newel pillar. Fig. 277.



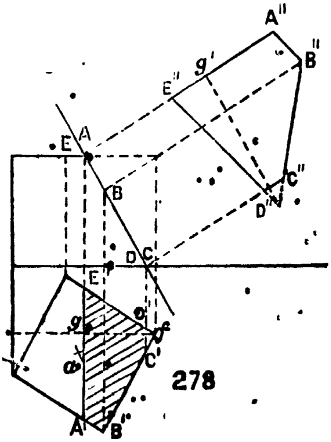
277

The preceding problems of helical curves explain the method of drawing the elevation of the round stair case. N is the newel of the staircase and the steps commence from *ab* in plan, the fronts of the first two steps are not seen in elevation. The lower helical curve gives the thickness of the steps. The front parts of the 3rd, 4th, 5th, 6th, 7th, steps in the first turn and 15th, 16th &c. in the 2nd turn are seen as rectangles. The intersection of the lower plane with the newel is seen in two places p. and q and obtained as helical curves. There are 12 steps in one turn. The steps 8, 9, 10, of the first turn and 20, 21

& 22 of the 2nd turn are completely hidden from view. It is a good exercise for the students.

Examples of sections :—

36. A cube of 1½″ sides stands on the horizontal plane, two of its vertical faces are inclined at 30° to the V. P. The cube is cut by a plane which is perpendicular to the V. P, makes an angle of 60° with the H. P. and passes through the centre of the top face of the cube. Draw the plan, elevation and true shape of section. Fig. 278.

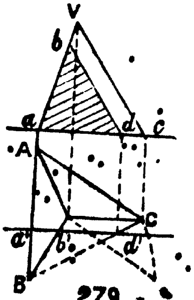


278

Sectional planes making angles with both the planes of projection fall under descriptive geometry problems and cannot be solved by the simple rules of projection. Here only those cases will be taken in which the sectional planes are inclined to one plane only and are at right angles to the other. The sectional plane can then be easily drawn as it will appear as a straight line on the plane to which it is perpendicular.

In problem 36 the sectional plane is inclined at 60° to the H. P and a line drawn from the centre of the top face in elevation at an angle of 60° with XY represents it. Obtain first the centre point *a* in plan and project it on the top line of the elevation as A. ABC is drawn at 60° with XY. Project the points A, B and C of the elevation on the plan The point A gives the two points A′ and E′, the projection from point B passes through the corner B′, and D′ and C′ obtained from C. A′B′ C′D′E′ is the plan of the section. The corner of the solid beyond the line D′C′ in plan and the portion on right of the section line ABC in elevation are dotted as they represent the portion cut off. For true shape of section draw a line from *f* in plan parallel to XY and mark the point *g* where it cuts A′E′. Draw in elevation a line *f′ g′* parallel to ABC the section line and project on this line from A, B, and C. Take measurements on the two sides of *f′ g′* equal to the distances of C′ D′ E′ A′ and B′ from *f g* in plan.. Join the points thus obtained and A″ B″ C″ D″ E″ is the true shape of the section whose plan is A′B′C′D′E′.

37. The plan of a tetrahedron is given (side 1 inch) resting on the H. P. on one face and the edge nearest to the V. P. makes 30° with it. Draw the elevation with the section made by a vertical plane parallel to the V. P. and ½ an inch from the nearest corner of the plan. Fig. 279.

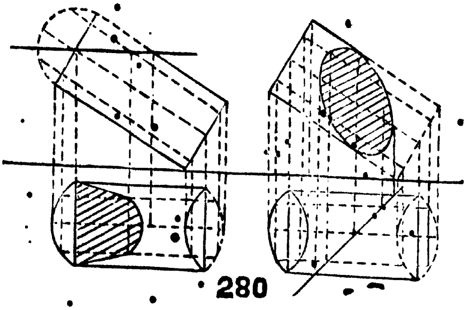


279

Draw an equilateral triangle. ABC represent the lower face of the tetrahedron with the side AC at 30° with XY; then the other side AB will be at right angles to it. Complete the plan of the tetrahedron and project its elevation. Here the sectional plane is a vertical plane and so is to be drawn as a

straight line in plan. The line a'b'd' in plan represents the sectional plane parallel to the V. P. Project the points a'b'd' on the elevation and the triangle abd in elevation represents the sectional part of the tetrahedron which is shaded.

38. A cylinder (dia. of base 1 inch ht. = 2") has its axis parallel to the V. P. It rests on the rim of its base which makes 60° with the H P. Draw plan and elevation when it is cut (1) by a horizontal plane passing through the middle point of the higher end. (2) when it is cut by a vertical plane making an angle of 45° with the V. P. and passing through the middle of the axis. Fig. 280.

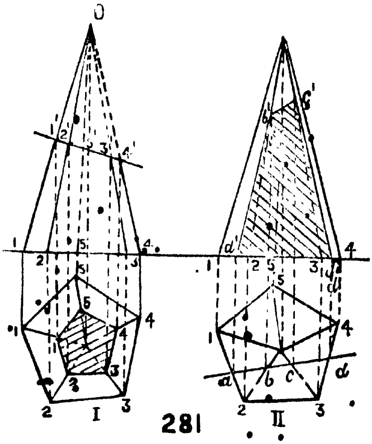

280

NOTE: Section of a cylinder cut by a plane perpendicular to the axis is a circle, and by a plane inclined to the axis, is an eliptical figure.

In case I. The section line is taken in the elevation and the sectional surface is seen on plan which is the true shape of section as the section line is parallel to the ground. The dotted semicircle on one end of the elevation is for obtaing lines on the surface to find points in the curve of section for projection. The inner curve of the eliptical fig. on the plan on the right is dotted, the outer one is firm.

In case II The section line is taken in the plan and the sectional surface is seen in the elevation reduced in length as the sectional plane is inclined to the V. P. The portions of the cylinder removed by the sectional plane is drawn dotted. The portions of edge lines of cylinder in plan on the right of the section line are dotted.

39. The plan and elevation of a pentagonal pyramid is given. It is (1) cut by a plane inclined to the ground. (2) cut by a plane inclined to the V. P. Draw plan and elevation of the sectional pyramid in the two cases. Fig. 281.

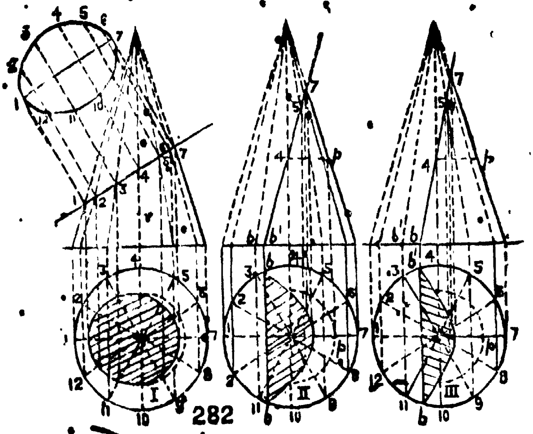


281

The two figures show the construction clearly. The plan of the pyramid a regular pentagon is drawn first. Fig. 80 part I gives an easy construction of a pentagon on a straight line. Where the sectional plane cuts the slant edges of the pyramid in the elevation of case 1 are projected on the respective lines in the plan. The point in the 5th line can not be projected correctly as it is very close to the axis of the figure. It is transferred by a horizontal line 5'p on the 4th line in elevation and the point p is projected in plan on the 4th line. It is transferred in plan by an arc of a circle on the 5th line.

Case II requires no explanation.

40. A cone stands with its base on the horizontal plane. Draw plan and elevation when it is cut by a plane (1) inclined to the ground at an angle less than the base angle of the cone. (2) when the cutting plane is parallel to a slant side of the cone (3) when the cutting plane makes a greater angle than the base angle of the cone. Show also the true shape of section of the 1st case. Fig. 282.



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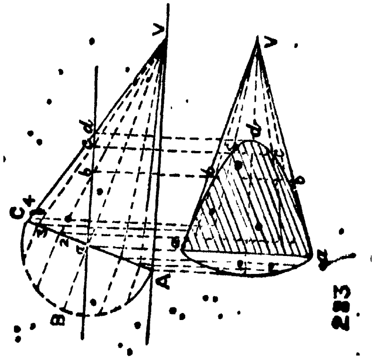
Case I. The section is an ellipse. Imagine 12 slant lines at equal distances on the surface of the cone cut by the sectional plane at 12 points in the curve of the section. These points are projected from the elevation where they are first obtained, on the respective lines in the plan and joining the points, the plan of the section is obtained. In plan the 4th and 10th points are obtained by projecting the point of the middle line in the elevation to one side as p and projecting it on plan and then transferring it by an arc of a circle to the middle line (4, 10) in plan. The true shape of section is obtained by drawing a line parallel to 17, the section line in the elevation and projecting on it the different points in the section line. Take distances on these projections on both sides of the line equal to the distances of the corresponding points in the plan from the line 17 in plan.

Case II. The section is a parabola. Part of the base is cut by the sectional plane. The points are first obtained in the surface lines in the elevation and then projected on the corresponding lines in plan.

Case III. The section is a hyperbola. Part of the base is also cut by the sectional plane. As the inclination of the sectional plane is greater than that in case II the plan appears thinner and in the maximum case it will be a straight line.

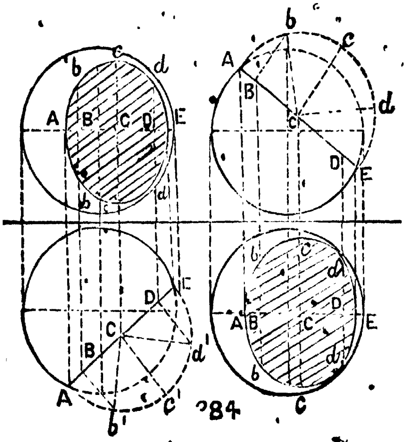
41. A cone lies on the ground on its slant side, it is cut by a horizontal plane passing through the middle point of the upper slant side. Draw plan. Fig. 283.



283

a d is the section line in the elevation. Draw a dotted semicircle on AC and divide the semicircumference into 6 equal parts and from these points draw perpendiculars to AC. Join these points with the vertex V for the surface lines which are cut by the sectional plane at a, b, c, d, project the points a, b, c, and d, on their respective lines in the plan. The points thus obtained in plan when joined gives the plan of the section and it is the true shape of the section as the sectional plane is horizontal.

42. Given plan and elevation of a sphere. It is cu by a plane (1) perpendicular to H. P. but inclined to th V.P. (2) cut by a plane inclined to the H. P. but perpendicular to the V. P. Fig. 284.

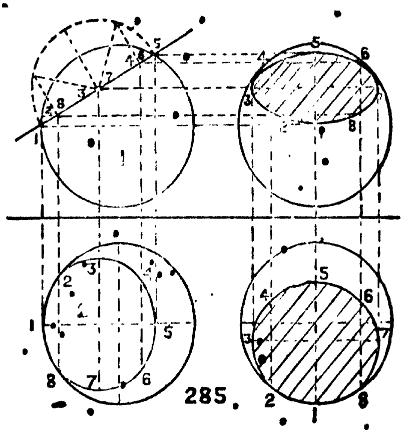


284

Case I. As the sectional plane is inclined to the V. P it is first drawn in plan cutting the sphere in the line A B C D E. Draw a semicircle on A E and divide the semicircumference into 4 equal parts at the points b', c', and d'. From these points draw perpendiculars on the section line AE meeting it at the points B, C and D. The points A and E are on the circumference of the plan which in elevation is the diameter of the circle. Project from the points A B C D E in plan on the diameter of the elevation and on these projections measure distances Bb, Cc Dd on both sides of the diameter equal to Bb' Cc' and Dd' of the plan. From near d to d in elevation the arc of the sphere is dotted. The sectional plane touches the circumference in points obtained by projecting from the point where AE cuts the dia. of the plan.

Case II. The sectional plane is inclined to the H. P. so it is first drawn in elevation as A E. Draw a semi-circle on A E for the points on the circumference of the section. Divide the semi-circumference into 4 equal parts at b, c and d and repeat the construction of case I. The circumference of the elevation is the diameter of the plan.

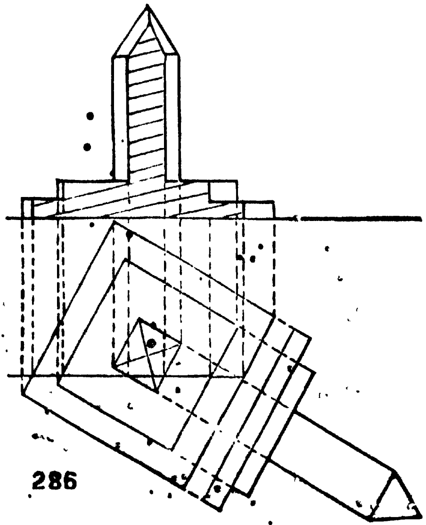
43. Given plan and elevation of a sphere; it is cut by a plane inclined to the H. P. at 30° and to the V. P. at 60° Fig. 285.



285.

Here the sectional plane is inclined in the front and can be easily drawn first in the side elevation in which case the plane appears as a straight line. No 1 is the side elevation of the sphere and the sectional plane is shown by the line 1-5. The plan of the section is obtained similarly to case I of the last problem. No. 2 is the plan of the side elevation that is of the sphere with the section turned 90°. The form of the section depends only on the inclination of the sectional plane to H. P. Turn No. 2, 90° to take its proper position as No. 3, and project from the points in the curve of section in No. 3 and the corresponding points of No. 1 for the points of No. 4 the elevation in its proper position. This is properly a case of Descriptive geometry solved by the rules of ordinary projection. In No. 4 the portion of the arc of circle (4, 5, 6) above the sectional plane is dotted.

44. Nos. 1 and 2 below XY figure 286 are the plan and elevation of a monument of masonry with two steps all round. Draw sectional elevation when the sectional plane is parallel to the V. P.

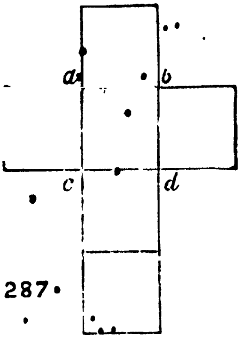


286

The plan No. 1 is placed so that the cutting plane A B is parallel to X Y. The points in the sectional elevation No. 3 are found by projecting from the points in plan No. 1 and measuring their respective heights from No. 2. The point d is projected on the elevation of the line hn from o the vertex of the pyramid. Parts of the right side of the monument appear beyond the sectional plane.

Examples of Development of surfaces of solids.

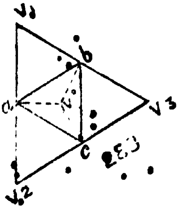
44. Develop the surfaces of a cube of 1" edge, Fig. 287.



287.

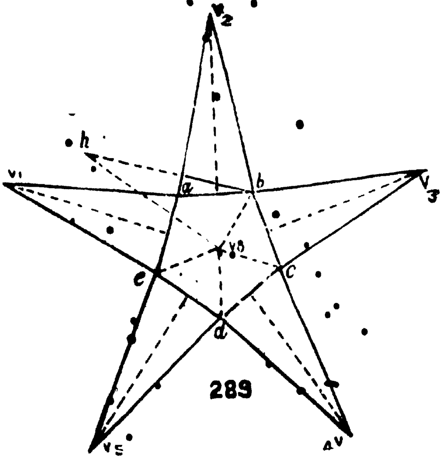
Draw *abcd* a square as the base of the cube. Produce *ab* and *cd* both, sideways and *ac* and *bd* both up and downwards. Draw one square on each side on the base *abdc* for the four-sides of the cube and one square below the lower square for the top. If the paper is cut on the outside lines and folded on the intermediate lines, a cube can easily be formed.

45. Develop the surfaces of a tetrahedron 1" edge. Fig. 288.



Draw an equilateral triangle *abc* for the base of the tetrahedron. On the three sides of the triangle draw 3 more equilateral triangles for the 3 faces of the tetrahedron. If the paper be folded on *ab*, *bc* and *ca* and the three equilateral triangles are raised by their 3 vertices V_1, V_2 and V_3 then they will meet at a point V_0 and form a tetrahedron.

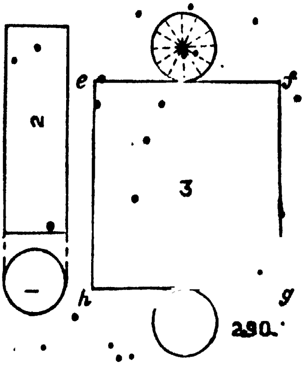
46. Develop the surfaces of a pentagonal pyramid edge of base ¾", height of axis 1½". Fig. 289.



289

Draw a regular pentagon *abcde* for the base of the pyramid. Find the centre of the pentagon v_0 and join v_0 with one corner *b* of the pentagon. Draw v_0h perpendicular to v_0b and make it equal to the height of the pyramid. Join hb. Then *hb* is the length of the slant edges of the pyramid. From two ends of each side of base draw arcs of radius *hb* intersecting in V_1, V_2, V_3, V_4, V_5, and join these points with the respective corners of the base. The five faces of the pyramid will be obtained. If these faces are raised on their bases their vertices will meet at a point the projection of which is v_0 the centre of the pentagon.

48. Develop the surfaces of a cylinder. Fig. 290.



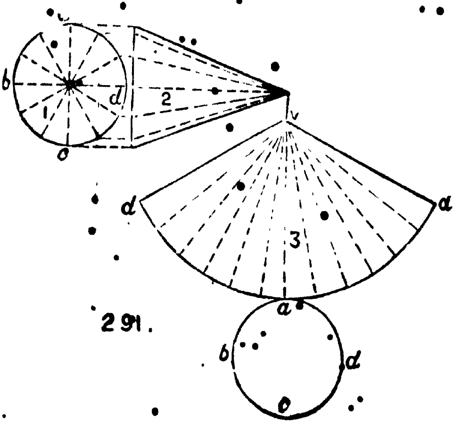
290.

The cylinder is given in plan and elevation as Nos. 1 and 2. Divide the circumference of No. 1 into 12 equal parts. Draw a straight line *hg* and measure on it 12 times a division of the circumference, then complete the rectangle *hf* for the development of the curved surface of the cylinder by taking the height *he*=height of the cylinder. Draw two circles of the diameter of No. 1 one touching *ef* the top line of the rectangle and above it and the other touching *hg* and below it for the top and bottom planes of the cylinder.

49. Develop the surfaces of a given cone: 291.

The cone is given in plan (1) and elevation (2). Divide the circumference of the plan, the base of the cone, into 12 epual parts.

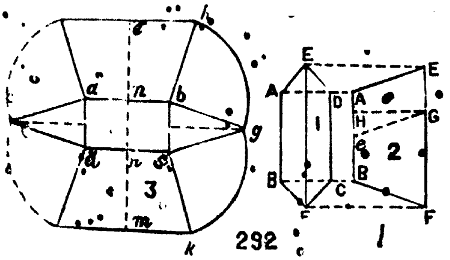
Draw an arc from any convenient point v as centre with radius equal to the slant side of the cone in elevation, and set off the 12 parts of the circumference of the base on this arc. Join the two last points with the centre. The sector *vdd* is the development of the curved surface of the cone. Draw a circle of radius equal to that of the base of the cone touching the arc and



291.

outside the sector, for the base of the cone.

50. **Develop the surfaces of a wedge, the two ends of which are isosceles triangles the length of its base is 1¼", breadth ⅝" and height 1". Its edge is 1¾" long. Fig. 292.**

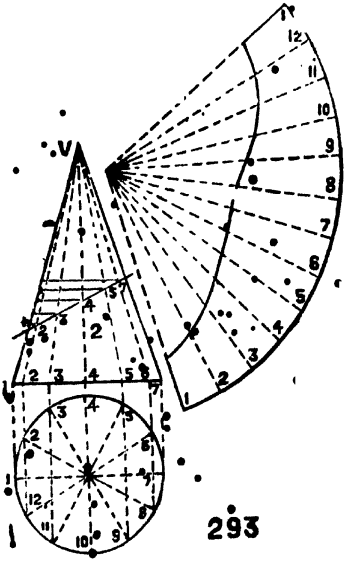


292

The plan and elevation of the wedge is given in Nos. 1 and 2. ABCD the base of the wedge 1¼" × ⅝" and the edge EF is 1¾".

long. The height of the wedge is GH shown in the elevation, it is not the real altitude of the trapezium AEFB inclined to the V. P. from its base AB. For the development draw a rectangle *abcd*=ABCD the base of the wedge. Take H*e* in AB No. 2 equal to half the width of the base *i.e.* $\frac{BC}{2}$, and join *eG*. From the middle points of *dc* and *ab* draw *pm* and *ne* at right angles to them and equal to *eG*. Draw parallells through *e* and m, equal to EF, and bisected by the points *e* and m. Join the ends with *a, b, c, d*. Draw arcs with these slant edges (*bh*) as radii from the points *b* and *c*, and *d* and *a*; join *bc* and *ad* with the points where the arcs intersect. Then the triangles thus formed as (*bcg*) are the end faces of the wedge.

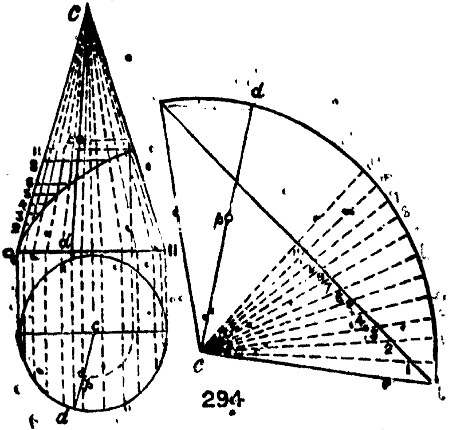
51. Show development of the curved surface of a cone with the section line of an ellipse. Fig. 293.



293

Draw plan (1) and elevation (2) of a right cone and draw on the elevation the section line of an ellipse. Draw a sector for the development of the curved surface of the cone with the slant side of the cone as radius and set off the 12 divisions of the circumference of the base of the cone on the arc of the sector. Join these points with the vertex of the sector. In the elevation (2) transfer the points 2, 3, 4, 5, and 6 on a side line of the cone for the true distances on the respective lines in the sector. Transfer these points on the respective lines in the development and by joining these points the development of the curve of the ellipse is obtained.

52. The development of the curved surface of a cone is given with the chord line drawn. Draw the plan and elevation of the cone showing the line. Find the length of the shortest line on the surface of the cone starting from a point in the base and coming back round the cone to the same point. Fig. 294.



294

No 1 abc is a sector; it is the development of the curved surface of a right cone; ab is joined. Bisect the arc of the sector and divide one of the halves into 11 equal parts. Take a line O11 (No. 2) and make it equal to 7 divisions of the *developed* arc. Take another line in plan equal and parallel to it. Draw a circle on this line as diameter. Project the two ends of the diameter on XY meeting it at O and 11. With these two points on XY as centres, and with bc or ac the radius of the sector as radius intersect arcs for *c* the vertex of the cone in elevation. Set off a division of the arc of the sector on the semi-circumference of the base, it will come exactly 11 times. Project these points on the base of the elevation, and join them with the vertex, for inclined lines on the surface of the cone. Measure 11, 22, 33 &c the distances of the points in the chord from the arc of the sector and place them first on the side line of the cone in elevation from the base, and transfer them to their respective places on the surface lines by lines drawn from these points, parallel to XY. The chord *ab* of the sector will then appear as a curve line found by joining the points last obtained. This is the shortest line on the surface of the cone from a point in base and coming round to the same point.

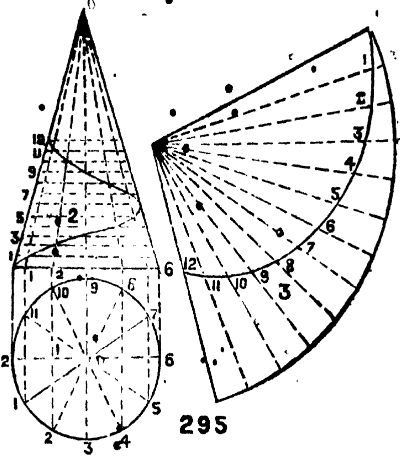
53. A point is given on the developed surface of a cone; find its position in plan and elevation of the cone. Fig. 294.

Let p be a point in the sector *abc* the developed surface of a cone. Draw plan and elevation of the cone by the preceding problem. Join c p in the sector and produce it to meet the arc in the point d. Take the distance of the arc b d on the base of the cone as o d and join d with the centre c of the base; project the point d on the base of the elevation as d and join d with the vertex c in the elevation. Measure d p in the sector and place it on a side of the cone in elevation from the base; transfer the point p' by a line parallel to XY on *dc* and the position of the point *p* in elevation is obtained. Project this point in the elevation on the line *dc* of the plan and the point p is found in plan.

54. Develop the surface of a cone with a helical curve on its surface. Fig. 295.

Draw plan and elevation of a cone as Nos. 1 and 2 and draw one turn of a helical curve on the elevation (2) by prop. 30

Fig. 272. Develop the surface of the cone and draw in the sector the 12 surface lines of the cone. Measure the distances

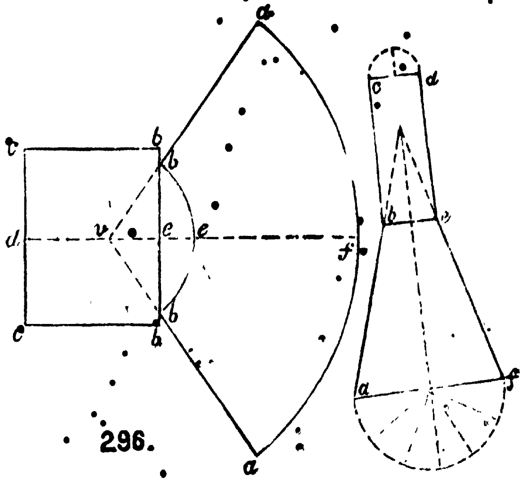


295

of the points of the helical curve transferred on one side line of the cone in elevation from the base of the cone No. 2. and place them on the respective lines in the sector, the development of the surface of the cone. By joining these points the development of one turn of the helical cvrue is found. (No. 3).

55. Draw the development of a tin funnel, the upper portion of which is frustum of a cone, the lower portion is a cylinder. Fig. 296.

The tin funnel is given in elevation as *abcdef*, on the line *af* of the mouth of the funnel draw a semicircle for the half rim of the mouth, and on *cd* the base of the cylindrical portion draw another semi circle. Divide the two semi-circumferences into 6 equal parts. Produce *ab* and *fe* the two sides of the mouth of the funnel to meet at *v* in No. 1 the elevation. With *va* as radius and from v any other point as centre draw an arc for the development of the frustum. From v draw vf the middle radius of the sector and set off on the arc 6 times on each side a division of the semicircumference of a f (No 1) to find a a the two ends of the arc. Join aa with v and measure a b on them equal to ab of No 1. With v as centre and v *b* as

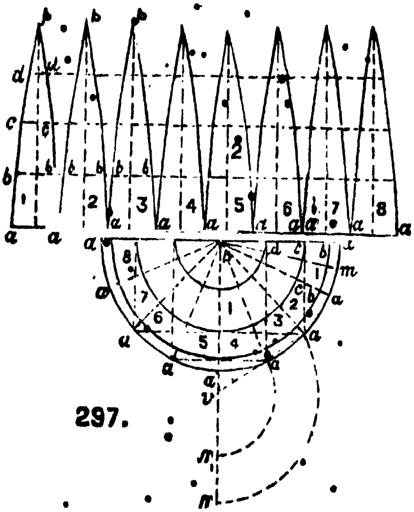


296.

radius draw another arc for the development of the other end of the frustum; join b b and produce. Let e be the middle point of *b b*. From e set off a division of the semi-circumference on cd (No 1) six times on each side of it, to find the outer points b and b Complete the rectangle *c b b c* for the development of the cylinder *bcde*.

56. Find the development of the surface of a sphere. Fig. 297.

A quarter of the sphere is shown by the semi-circle *a a a* (No 1). Divide the arc of the semi-circle into 4 equal parts in a, a, a, &c and draw from these points lines perpendicular to the diameter for the points b, c, d, &c. Let p be the centre of the semi-circle or the pole of sphere. With p as centre and radii *pb*, *pc* and *pd* draw 3 more semi circles for the plan of the 4 divisions on the surface of the quarter sphere. Bisect the division a a of the circumference in *m* and join *m* P. On a straight line set off 8 spaces each equal to one distance *a a* of the circumference and bisect each of these divisions. From these points of bisection draw lines perpendicular to the



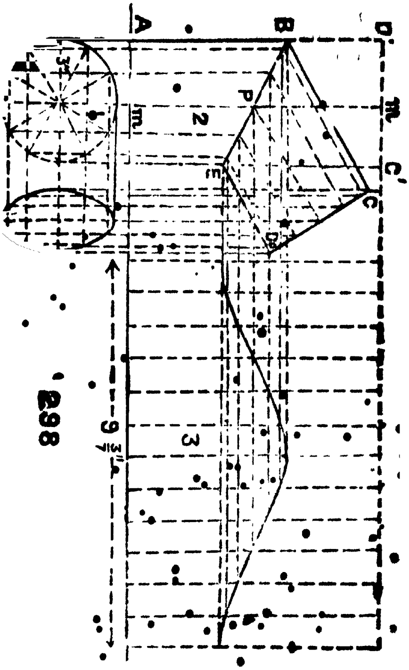
297.

straight line; on these perpendiculars set off the distance a a 4 times which gives the development of the lines *m p* on the surface of the sphere.

Draw three lines parallel to the base through the three intermediate points on the perpendicular, and take half the distance of *bb*, *cc* and *dd* and place them on the two sides of the perpendiculars on the three lines, to get the points b c d on each side of the perpendiculars. Name the top point as p. Join *p d c b a* on the two sides of each perpendicular. If these 8 semi-lunes be joined on their curve lines bringing all the points p together a quarter sphere is obtained. This is dividing the sphere by *lunes*. To obtain the division of the surface by horizontal slips as zones produce *pa* the vertical line in No 1 to n. Join the two points of the 3rd zone a_3, a_4 and produce to meet *pa* produced in v. With v as centre and

va_8 and va_4; as radii draw two arcs to meet pa produced in n and n′ and then $a_8\ n\ n'\ a_4$ is the development of quarter of the 3rd zone from the middle.

57. A cylindrical pipe turned at an angle of 120° is to be formed out of a sheet of metal measuring 6¼″×9¾″. The arms of the pipe are to be equally long. Show how the sheet metal is to be cut. Fig. 298.



298

Draw a rectangle abcd with side ab = 9¾″ and side bc = 6¼″ Divide ab into 12 equal parts and from the points draw lines parallel to *cb*. For the plan of the cylinder draw a straight line equal to $\frac{9\frac{3}{7}''}{3\frac{1}{7}} = 3''$ parallel to xy, and draw a circle on it as diameter. Set off a division of ab 12 times on this circumference and draw the elevation of the cylinder with the lines on its surface. Bisect the middle line m m at p and through p draw a section line BE at 30° with XY. Transfer the heights of the lines on the surface of the cylinder in the elevation intercepted by BE to their respective places in the rectangle which when joined will give the development of the section line. The upper portion of the cylinder is turned 180° on its axis in elevation to obtain the bent of 120°

CHAPTER III.

ISOMETRIC PROJECTION.

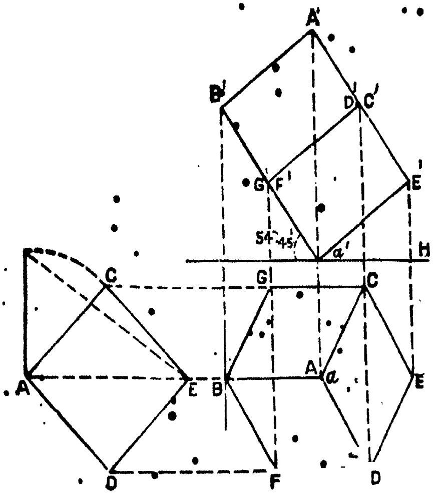
The principles of isometric projection enable the three dimensions of a solid to be shown by one drawing, which, in appearance, is somewhat similar to a perspective representation with the additional advantage that the actual size of the solid can be measured direct from the drawing.

(This system of projection was discovered by Professor Farish of Cambridge, about the year 1820.)

It is well suited to show the forms and dimensions of all rectangular solids or solids bounded mainly by three systems of planes mutually perpendicular, such as examples of wood work &c. For this projection only one drawing is required instead of several.

In Isometric projection, which is a particular case of ordinary projection, the object is always in a fixed and constant position with regard to the plane of projection i. e. the H. P. This position is such that the three principal axes or edges of the object (as length, breadth and height of a rectangular prism) shall be equally inclined to the H. P.; and all straight lines parallel to them are drawn in proportion to the same scale. Its principle is based on the projection of a cube. If a cube be made to rest on one corner upon the paper, so that a diagonal of the solid is vertical, its plan will be represented as shown in the following figure.

The three edges of the cube (AB,AC,AD) terminating at one end of the solid diagonal (A) are all equally inclined to the ground and consequently are equal in length in plan and make equal angles between themselves (120°). The three top faces of the cube are ABGC, ABFD and ACED. The three lower faces are aGCE, aFDE and aGBF. As the plans of the three top faces are similar and equal figures, they are equally inclined to the ground. All the other lines of the cube are parallel and equal to one or the other of the three edges AB, AC, AD. The figure is easily constructed as the lines BG and BF make angles of 30° with the projector from B′. Draw a line through B in plan perpendicular to XY which is the starting line of the figure and BA the edge which is at right angles to the base aGBF is perpendicular to this line. The



299.

figure starts from the point B which is in space and not from a which touches the ground. The lines which are parallel to BF and BG i. e. the lines forming the tops and bases of the cube are drawn parallel to them, and the lines representing the heights of the cube as GC, FD, aE are drawn parallel to BA. The base of the cube is inclined at 54°-45′ to the ground.

The figure can easily be drawn by two set squares one of which must have 60° and 30° angles. Fig 299.

The above reasoning only strictly applies to oblong solids, having solid right angled corners, but the same construction can be very conveniently applied to irregular solids and solids with curved surfaces.

Isometric Scale :—

Referring to the preceding projection of the cube it will be seen that the lengths of the edges of the cube in the isometric view can not be equal to their real lengths as they are all inclined to the ground.

The projected length of the edge $a'E' = a'H = aE$ in plan

i. e. $\frac{\text{actual length of an edge}}{\text{its projected length}} = \frac{a'E'}{aE} = \frac{a'A'}{A'E'} = \frac{\text{solid diagonal of a cube}}{\text{face diagonal}}$

∴ The two triangles $E'a'H$ and $E'A'a'$ are similar as the $\angle A'E'a'$ and $\angle E'Ha'$ are right angles and $\angle E'A'a' = E'a'H$.

But $a'E' =$ edge of cube say $= 1$.

$E'A' =$ diagonal of a face $= \sqrt{2}$

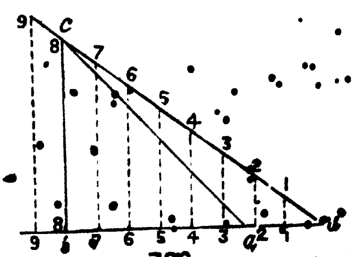
and $a'A' =$ solid diagonal of cube $= \sqrt{3}$

∴ $\frac{\text{Actual length of a line}}{\text{its isometric projection}} = \frac{a'E'}{aE} = \frac{a'A'}{A'E'} = \frac{\sqrt{3}}{\sqrt{2}}$

The reduction in the scale of the isometric is $\frac{\sqrt{2}}{\sqrt{3}} = \cdot 816$ nearly.

1st method of drawing isometric scale. Fig. 300.

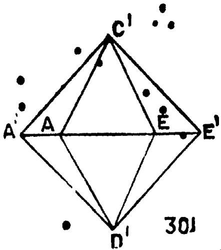
Draw a line *ab* of any length, at one end *b* of *ab* draw a perpendicular *bc* equal to *ab*. Join *ca*. Then *ca* is the face diagonal of a cube. Produce *ba* to *d* making $bd = ac$ and join *dc* then *dc* is the solid diagonal of the cube. Then $\frac{dc}{db} = \frac{\sqrt{3}}{\sqrt{2}}$ i. e. the isometric of *dc* is *db*. Set off any scale on *dc* and from the different divisions draw lines perpendicular to *db*, then the divisions on *db* will be the isometric scale. The scale on *cd* is the natural scale and that on *bd* is the isometric scale, Fig. 300.



300

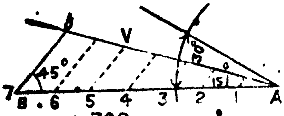
2nd Method of drawing isometric scale :—

The square face A′C′E′D′ of the cube is projected isometrically as AC′ED′ a rhombus i. e., the line A′C′ is projected as AC′ in the marginal figure. The diagonal C′D′ and distances parallel to it remain unchanged and the diagonal A′E′ is shortened to AE in the ratio of $\frac{\sqrt{3}}{1}$: 2 greater reduction than the isometric reduction of $\frac{\sqrt{3}}{\sqrt{2}}$. It is called the minor scale. The isometric length of A′C′ = AC′. The angle ∠C′A′A=45° and ∠ A′C′A=(45−30)=15°.



301

Draw a line AB of any length. From one end A draw a line at an angle of 15° and from the other end B, another line at 45° with AB; the two lines meet at b. Then the isometric of AB is Ab. Set off any scale from A on AB and from the different divisions of it draw lines parallel to Bb then the divisions on Ab is the isometric scale. Fig, 302.



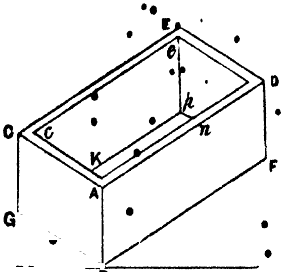
302

Isometric Projection of a cube is given in fig. 299 and isometric scales by 2 methods are shewn in figs. 300 and 302.

NOTE:—Isometric scale is not ordinarily used: The figure is drawn by taking measurements from the natural scale; its effect is to make the figure appear larger but the similarity of form is retained.

58. Isometric projection of a box without a lid. Fig. 303.

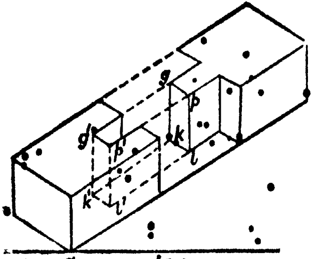
Draw the outside of the box ABFDECG by moving and turning a 30° set square in contact with the long edge of 45° set square, placed with that edge horizontal, and touching the starting point. Place the thickness of planks on the top edges from the four corners A D E C and draw the inner lines ce &c. Draw a line eh perpendicular to the horizontal line as B from e and make h equal to the inner height of the box, which is less than the outer height, by the thickness of the bottom plate. Draw hk and hn



303

parallel to the two sides of the box for the sides of the bottom plate.

59. Isometric projection of a rectangular piece of wood (made up of two similar pieces joined by mortise and tenon joint). Fig. 304.

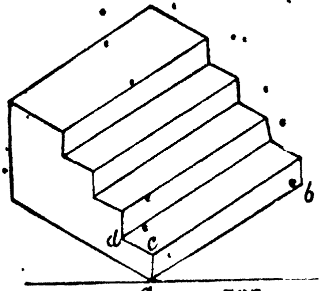


304

The two parts of the rectangular piece of wood are drawn with the joint opened out. The figure is easily drawn by a 30° set square starting from the point a. The face gklp of the tenon agrees to the back of the mortise g'k'l'p'.

60. Isometric projection of a flight of steps, Fig. 305.

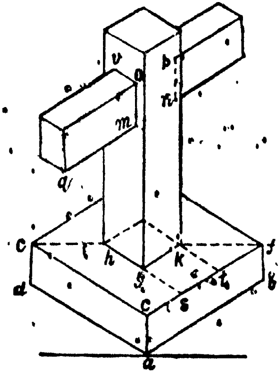
Three steps and one landing for the top are shown in the figure. The width of a tread *dc* is greater than the rise *ac* of a step. Draw *ab* the length of step in the right hand isometric plane; draw *ac* at right angles to the horizontal start line and complete the rise. Draw *cd* the width of a tread from c in the left hand isometric plane and complete the tread. The other treads and landing are drawn similarly. The figure is a simple one.



305.

61. Isometric projection of a cross resting on a pedestal. Fig. 306.

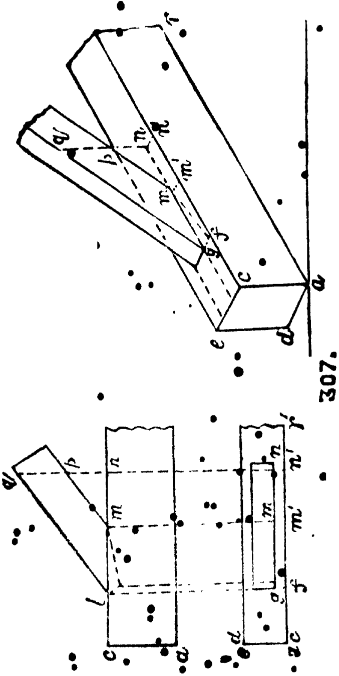
The shaft of the cross is to be placed centrally on the square pedestal The arm of the cross is rectangular in section, its thickness is reduced to make it pass through the hole in the shaft which is square in section First draw the isometric of the pedestal and join the horizontal diagonal ef of its upper face. Take the two distances es and ft each equal to half the difference of a side of the pedestal and a side of the shaft and draw sh and tk parallel to ce to meet the diagonal ef in h and k Measure hg equal to a side of the shaft, then gk joined is equal to hg and is the other side of the base of the shaft. Complete the shaft. Draw mo and ov, the two sides of the hole on the left or on the right



306

side of shaft as the arm penetrates the left or the right face. Draw the isometric of the arm on these two lines. The line qm is produced to r, the extreme right point of the line. The distance mn = gk and nr = qm.

62. Isometric projection of the joint of tie-bar with the principal rafter. Fig. 307.



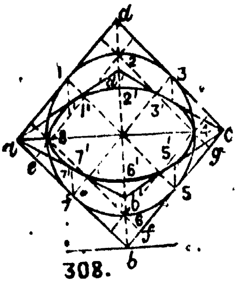
307.

The joint is shown by its plan and elevation. Draw the the isometric of the tie-bar and place on it distances fm' and m'n' from the plan. h is the foot of the perpendicular on the tie-bar from the two points q and p in the same vertical line on the two edges of the principal rafter for drawing it isometrically. Only lines parallel to the three axes of a cube can be drawn isometrically For the representation of inclined lines on isometric planes the rule of right-angled triangle is to be adopted for making their resolved parts parallel to the two axes of a cube.

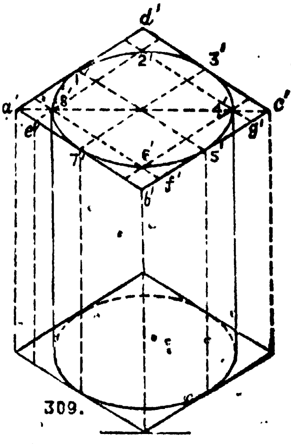
Draw gmn and npq isometrically with their respective measurements from the plan and elevation. Join gq and mp which gives one face of the rafter. Complete the other face seen.

63. Isometric projection of a circle and a cylinder. Figs. 308; 309.

Draw a circle 1357 fig. 308 and describe a square abcd about it with its two sides ab, bc at 45° with the horizontal direction.



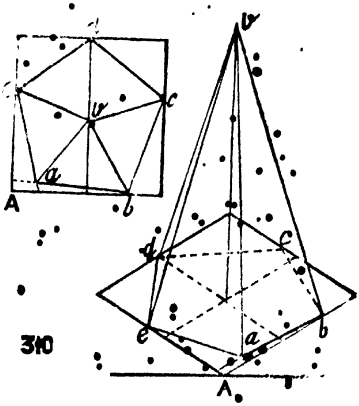
308.



309.

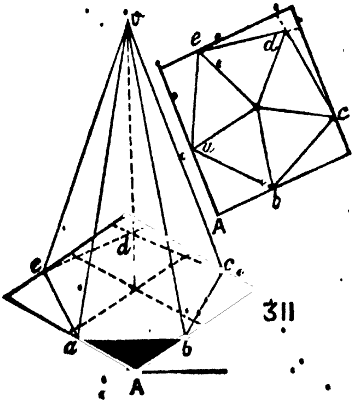
Join the diagonals ac, db fig. 308 and through the points where these diagonals intersect the circumference, draw lines parallel to the sides of the square The 8 points of the circumference, the four touching points and the four diagonal intersections are obtained for the points of the curve (ellipse) in the isometric. In fig. 309 draw a' b' c' d' the isometric projection of the square abcd. Find in the isometric square the middle or touching points and the diagonal intersections of the curve. Number these points as 1', 2', 3', 4', &c. By joining these points the isometric view of the circle is obtained. The portion 1' 2' 3' of the curve is an arc of a circle with b' as centre and b' 1' as radius and similarly the portion 5', 6' 7' is an arc of a circle with d' as centre. The cylinder (fig 309) is finished by drawing two tangents on the two sides of the ellipse and making their lengths equal to the height of the cylinder. The base of the cylinder is drawn similarly to the top, only half the curve is seen. The ellipse 1' 2' 3' 4' etc. fig. 309 is bigger in size than to its original the circle 1357 as the isometric view of the square abcd is not reduced to the isometric scale. In the square abcd (fig. 308) draw its isometric a'b'c'd' and draw the ellipse inside it by finding the 8 points of the curve. This ellipse is the true representation of the circle on the isometric plane, as the line ab' is the isometric of the line ab.

64. Isometric projection of a pyramid and a cone. Figs. 310, 311 and 312.



310

A pyramid can conveniently be drawn in isometric by enveloping it in a rectangular prism of the same height as the pyramid. A corner of the base of the pyramid must lie on a side of the base of the prism. Pentagonal pyramid is taken in the example and two ways of drawing the pyramid (one edge in front and one face in front) are shown in figs. 310 and 311.



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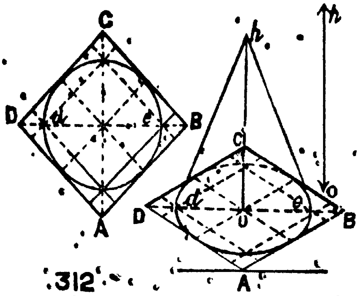
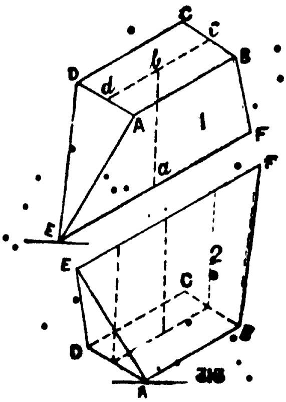


312

Fig. 312. Isometric projection of the cone is obtained by imagining the cone to be enveloped in a cylinder of the same base and height. In fig. 312 the base of the cylinder is drawn in isometric and the height is placed as a vertical line from o the centre of the ellipse. The line oh is equal to the height

of the cone. From h the vertex of the cone or the centre of the top face of the cylinder draw tangent lines to the ellipse of the base. The tangents touch the curve at points nearer than d and e the ends of the major diameter. A little greater than half the cone is seen in the isometric view. In case of cylinder exactly half of the cylinder is seen in the isometric projection.

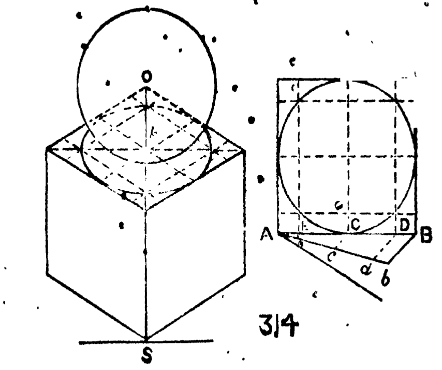
65. Isometric projection of a wedge-shaped piece of wood. Fig. 313.



313

A wedge similar to one given in figure 292 (Nos. 1 & 2) is drawn isometrically in fig. 313. Two views are given, 1 with the edge below and 2 with the edge upwards. In 1 draw EF the edge line first isometrically. Bisect it at a; draw ab, the height vertically and draw dc through b, parallel to EF, making db and bc each equal to half the length of the base of the wedge. Through d and c draw DA and CB the sides of the base. Join AE, DE and BF. In 2 the base is drawn isometrically first. Bisect the two sides of the base and draw the middle line of the base. From the two ends of this middle line draw verticals equal to the height of the wedge; through the tops of these verticals draw EF the edge. Join AE, DE and FB. 1 gives a better view than 2.

66. A cube of 1″ edge with a circle inscribed on the top face and supporting a sphere of diameter 1″ on its top in contact with the centre. Draw the isometric view of the cube with the sphere on it. Fig. 314.

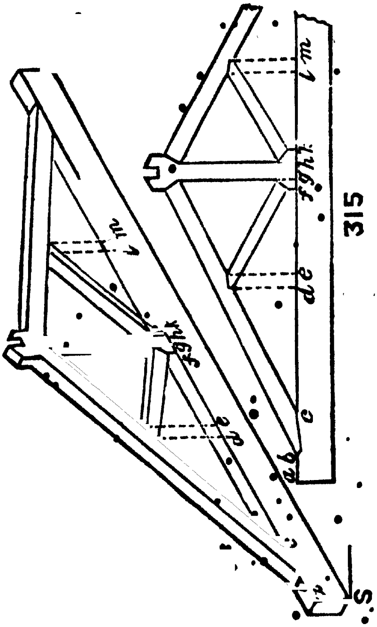


314

There was no need of using the isometric scale for the previous figures. They are all drawn from the natural scale and the figures increased proportionately in all directions. The same reasoning is not applicable to this case as a sphere retains its full size and form in whichever way it is raised and turned. The sides of the cube will be reduced in the isometric view and if both the cube and the sphere be drawn on the natural scale, the sphere remains the same, but the cube is much enlarged and the proportion of the combination is altered.

Draw a square of side equal to an edge of the cube and draw a circle inside it. Draw the diagonals and the other lines for the 8 points of the circle. Draw Ab the isometric of AB, and transfer the measurements AE, EC, CD from the natural line to the isometric. Draw the isometric of the cube starting from S with the sides = Ab the isometric length. Draw the ellipse on its top face representing the circle there. Find the centre p and draw pO at right angles to the base line from p and equal to *Ac* the height of the centre of the sphere reduced isometrically. With O as centre and radius equal to AC the radius of the sphere from the natural scale draw a circle which will represent the sphere touching the top plane of the cube at p.

67. Isometric projection of a king post roof-truss. Fig. 315.



315

The truss is shown below by its elevation. Draw the tie-bar in isometric first and get on its upper edge the distances of the points shown in the elevation. The rafters are drawn similarly to fig. 307 and the struts are drawn also by offsets.

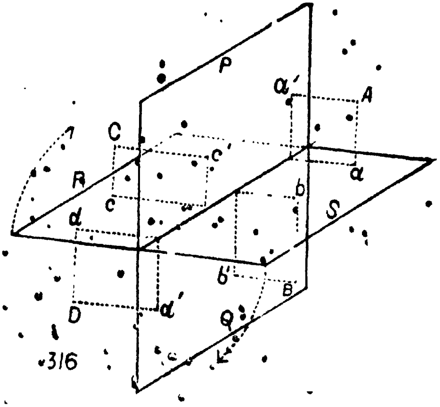
CHAPTER IV.

DESCRIPTIVE GEOMETRY.

The principal object of Descriptive geometry is to represent solid figures on a plane surface and the graphic solution of the problems of solid geometry.

Pure solid geometry treats geometrically the relations which exist amongst points, lines and surfaces in space but by practical solid geometry we can show on paper, which has only two dimensions, forms of figures in space of three dimensions in such a manner that the positions and forms can be ascertained from the drawings.

Like orthographic projection the objects in this case are projected perpendicularly to two co-ordinate planes which are considered as indefinite in extent. The H.P. is extended beyond the V. P. and the V. P. below the H. P. (Fig. 316.)



316

The combination of co-ordinate planes used in descriptive geometry is shown in fig. 316, which shows four sets of co-ordinate planes, or as they are called "dihedral angles."

The first set of co-ordinate planes formed by P, the front of the vertical plane with S, the upper surface of the H. P. and in front of the V. P is called the "first dihedral angle."

The second set of co-ordinate planes formed by P, the back of the vertical plane with R, the upper surface of the H. P. behind the V. P. is called the "second dihedral angle."

The third set of co-ordinate planes formed by Q. the back of the lower vertical plane with R, the under surface of the H. P. behind the V. P. is called the "third dihedral angle."

The fourth set of co-ordinate planes formed by Q. the front of the lower vertical plane with S, the under surface of the front portion of the H. P. is called the "fourth dihedral angle."

(1) A is a point in the first dihedral angle and a and a' are its plan and elevation.

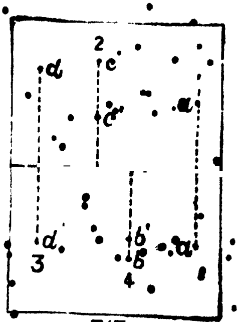
(2) C is a point in the second dihedral angle and c and c' are its plan and elevation.

(3) D is a point in the third dihedral angle and d and d' are its plan and elevation.

(4) B is a point in the fourth dihedral angle and b and b' are its plan and elevation. (Fig. 316.)

The line where the two co-ordinate planes cut is always named XY and called the ground line.

If the horizontal plane R S is turned as shown by the circular arrow-head it coincides with the vertical plane P Q' and fig. 316 appears as fig. 317.



317

Fig. 317. No. 1. The projection is seen in front of the

vertical plane P Q with XY as ground line ; the plan *a* appears on the lower side of XY which is the horizontal plane and the elevation *a'* is shown on the upper side, the vertical plane.

In no. 2 the horizontal plane coincides with the V. P and the plan and elevation of C appears on one side of XY *i. e.* in the vertical plane but behind it and so cannot be seen.

In no. 3 the horizontal plane coincides with the upper vertical plane, the plan d of the point D is seen on the elevation and the elevation d' is seen on the lower side of XY which is the vertical plane for cases below the H P. The points all appear behind the V P and cannot be seen.

In no. 4 the horizontal plane coincides with the lower vertical plane, the plan and elevation of the point B appear on one side of XY *i. e* the lower side of it, and are covered by the horizontal plane and so cannot be seen. Only the points of the first case are visible.

From fig. 317 it will be seen that the perpendiculars drawn from the projections of a point to the ground line meet it in the same point. This is the same as no. 4 of the rules of projection.

The problems of descriptive geometry are made easy by finding out the traces of lines or planes with the co-ordinate planes.

Traces :—The point where a line in space or the line produced meets the horizontal plane is called the horizontal trace of the line, and the point where it meets the V P is called the vertical trace of the line. The line of intersection of a plane or a plane produced with the horizontal plane is called the horizontal trace of the plane and the line in which it intersects the V P is the vertical trace of the plane.

Certain theorems in solid geometry are self-evident and may be taken as axioms in plane geometry.

1. Two straight lines which cut one another are in one plane.

2. If two planes cut one another, their common section is a straight line.

3. If a straight line be perpendicular to each of two straight lines at their point of intersection, it shall also be perpendicular to their plane.

4. Every plane which contains the normal to another plane is perpendicular to that plane.

5. If two planes which cut one another be both perpendicular to a third plane, their common section shall be perpendicular to the same plane.

6. Two straight lines which are perpendicular to the same plane are parallel to one another.

7. Two straight lines in space which are each parallel to the same straight line are parallel to one another.

8. If a straight line be parallel to a plane, it shall be parallel to the line in which any plane containing it cuts the first plane.

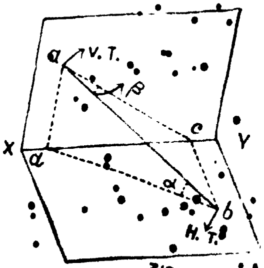
9. If two parallel planes be cut by another plane, their common sections with it shall be parallel.

10. Planes to which the same straight line is perpendicular are parallel to one another

11. The projection of a straight line on a plane is a straight line, because it is the intersection of a plane, containing the straight line and perpendicular to the plane of projection with the plane.

12. If two straight lines be at right angles to one another, their projections on a plane parallel to any one of them shall also be at right angles.

To find out the angle which a line makes with a plane :—Let one end of the line touch the plane and from the other end or any point of the line, draw a perpendicular to the plane; join the foot of the perpendicular with the trace or the point where the line touches the plane. The angle which the given line makes with this line is the angle with the plane. (Fig. 318.)



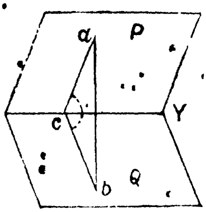
318

a is the vertical trace of a line and b the horizontal trace, from b draw bc perpendicular to the vertical plane and join a the

VP with c. The $\angle$ bac *i. e.* $\angle \beta$ is the angle which the line makes with the VP. Similarly $\angle$ abd *i. e.* $\angle \alpha$ is the angle which the line makes with the HP.

To find out the angle, which a plane makes with another plane. In the common section of the two planes take a point and from that point draw two lines, one in each plane perpendicular to the line of intersection of the two planes, then the angle between the two perpendiculars is the angle between the two planes.

Let P and Q be two planes meeting at an acute angle. Take any point C in XY the common section of the two planes, draw ca and cb perpendiculars to XY in planes P and Q respectively. The $\angle$acb in the plane acb is the angle between the two planes. (Fig. 319.)



319

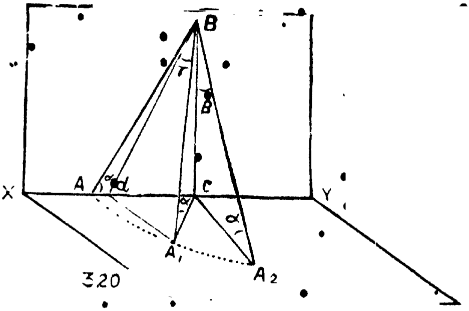
A line can make with a plane any angle from 0° to 90° and the sum of angles which a line makes with the two planes of projection can not be more than 90 .

The sum of the angles which a plane can make with the two co-ordinate planes is in the least 90 and can not be more than 180°. It is the least when the plane is perpendicular to one of the co-ordinate planes and parallel to the other and greatest when it is at right angles to the VP as well as to the HP.

AB is a line inclined at an angle α with the HP and placed touching the vertical plane as AB. From B draw BC perpendicular to the ground line XY and with C as centre and CA as radius draw quadrant of a circle AA_1 on the HP. Join A_1B. Then the angle CA_1B is α and $\angle$ CBA_1 is the angle which the line AB makes with the VP *i.e.*, $\angle \beta$. (Fig. 320.)

$$\angle\alpha + \angle\beta = 90^\circ \text{ as } \angle BCA_1 = 90^\circ.$$

$\angle\beta$ is the greatest angle which a line inclined to the HP at $\angle\alpha$ can make with the VP. Take another position of the right-angled triangle BCA as BCA_1; the line A_1C make 45° with XY.

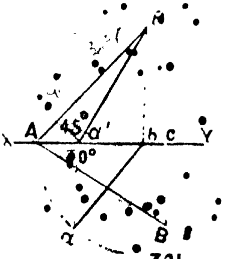


320

The $\angle BA_1C$ is $\angle\alpha$. But $\angle A_1BC$ is not the angle which A_1B makes with the VP. From A_1 draw A_1d at right angles to XY. Join dB. Then the angle dBA_1 is the angle which A_1B makes with the VP.

Problems.—

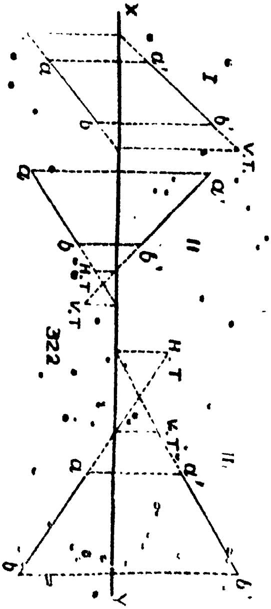
68. Determine the projections of a line 2″ long making an angle of 45° with the HP and 30° with the VP. (Fig. 321.)



321

The construction is similar to fig. 252 prob. 17. First the lengths Ab and Ac plan and elevation respectively of the line when it is inclined at 45° with the HP and 30° with the VP, separately, are found. Then these two lengths are to be placed in their respective positions so that one can be projected from the other. Imagine the line first placed touching the VP as AB', showing its inclination with the HP. The line can then be turned either keeping the end A fixed or the end B' fixed. In the figure the end A moves.

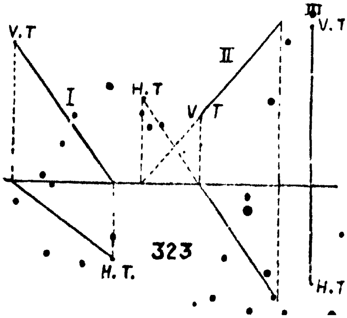
69. Given the projections of lines, to find their traces. 3 cases. (Fig. 322.)



322

Produce the elevation of the line to meet XY, and from the point, draw a line perpendicular to XY, and where this perpendicular meets the plan produced is the HT of the line. Similarly draw a perpendicular from the point where the plan produced meets XY, and where this perpendicular meets the elevation is the VT of the line. In case II the plan meets XY beyond the point where the elevation meets it, so the elevation is produced to find the VT which is in the 4th dihedral angle. In case III the elevation meets XY beyond the point, where the plan meets it so the plan is produced to find the HT in the 2nd dihedral angle.

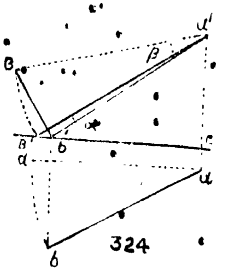
70. To find the projections of a line, its traces being given. Fig. 323



323

From the two points HT and VT of a line which are given, draw perpendiculars to XY. The elevation of the line is found by joining the point, where the perpendicular from the HT meets XY, with the VT of the line, and the plan, by joining the HT of the line with the foot of the perpendicular from the VT. In case II the projections of the line are found by producing the lines thus obtained. In case III the HT and VT are so given that the perpendiculars from them to XY are the projections of the line. In this case the line is in a plane which is at right angles to both the horizontal and the vertical planes.

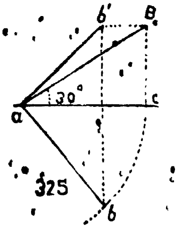
71. The Projections of a line being given, to find its inclination to each plane and its true length. Fig. 324.



324

a'b' is the elevation and *ab* is the plan of the line. With *a* as centre and *ab* as radius draw an arc *bd* to meet the horizontal line *ad* from *a*. Then the plan is put parallel to the vertical plane. Draw the ground line through *b'*. Project from *d* to meet the ground line at B'. Join B'a' which is the true length of the line. Then a'B'C is the angle which the line makes with the horizontal plane i e, ∠α From b' draw b'B perpendicular to *a'b'* the elevation. From *a* as centre and with a'B' as radius draw an arc to meet b'B in B. Join a'B. Then Ba'b' is the angle which the line makes with the vertical plane i e., ∠ β.

72 The horizontal projection of a line and the angle it forms with the HP are given to find its vertical projection. Fig. 325.



325

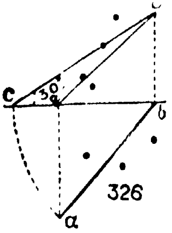
ab is the horizontal projection of a line and 30° is the angle which it makes with HP. With *a* as centre and *ab* as radius draw the arc *bc* to meet XY in c. From *a* draw ab at 30° with

XY. From c draw cB perpendicular to XY to meet aB in B. Then aB is the length of the line whose plan is *ab* In turning *ac* to the position *ab* the height Bc is unchanged. From B draw Bb' parallel to XY. Project from *b* to meet *Bb'* in *b'*. Then *ab'* is the elevation of the line.

98 The vertical projection of a line and its angle with the HP are given to find its horizontal projection. Fig 326.

a'b' is the vertical projection of a line which is inclined at 30° with the HP Draw from *b'* a line b'C inclined at 30° with XY Draw *b'b* perpendicular to XY. Then bC is the length of the plan of the line With b as centre and bC as radius draw Ca an arc. Draw a line perpendicular to XY from a' to meet the arc in a. Join *ba* then *ba* is the plan.

99 The plan and elevation of an angle being given to find its traces and the true angle. Fig. 327.



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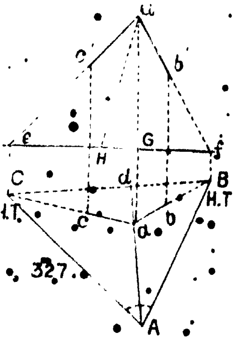
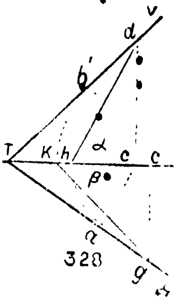


327

c'a'b' is the elevation and *cab* is the plan of an angle. Produce *a'c'* and *a'b'* to meet the ground line in e and f Project from e and f to meet *ac* and *ab* produced in C and B respectively. C and B are the horizontal traces of the angle. From *a* draw *ad* perpendicular to BC. From á in elevation draw áG perpendicular to XY. Take GH equal to *ad* and join áH. Produce *da* and take dA in it equal to áH. Join AC and AB. Then the angle BAC is the true angle obtained by throwing the angle on the horizontal plane by turning on BC.

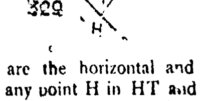
100 From the H. T and V. T. of a given plane, to determine the angles it forms with the two planes of projection. Fig 328.



328

TH and TV are the horizontal and vertical traces of a plane In TH take any point *a* and draw *ac* perpendicular to TH to meet XY in *c*. From c draw *cd* perpendicular to XY to meet the vertical trace in *d* Take *ch* in XY equal to *ca* and join *hd*. The ∠ chd is the angle which the plane makes with the H.P. Similarly the angle c'kg is the angle which the plane makes with the V P.

101. To find the true angle between the H T. and V.T of a given plane. Fig. 329.



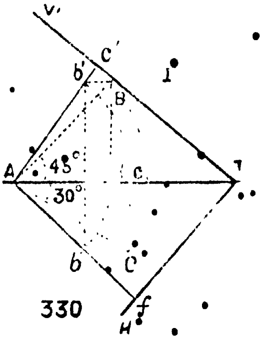
329

HT and VT are the horizontal and vertical traces of the given plane. Take any point H in HT and draw Ha perpendicular to HT meeting XY in *a* From *a* draw aV perpendicular to XY to meet the vertical trace in V. Produce Ha and from T as centre and TV as radius draw an arc to meet Ha produced in v′. Join Tv′. Then <HTv′ is the true angle between the HT and VT of the plane.

102. Determine the traces of a plane inclined at 45° (α) with the H.P. and 60° (β) with the V.P.

I By the projection of a line perpendicular to the plane Fig 330.

II By the conical method Fig 331



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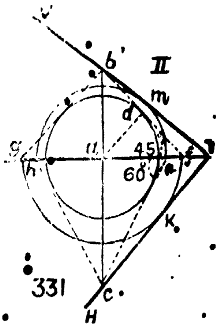


331

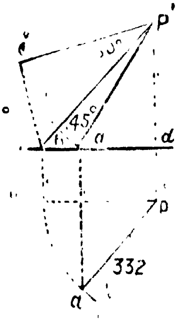
Theorem :—If a line be perpendicular to a plane, the plan and elevation of the line are respectively perpendicular to the horizontal and vertical traces of the plane.

I. Fig 330 Draw plan and elevation of a line perpendicular to the plane and inclined at 45° (90 α) with the HP and at 30° (90 β) with the VP (prob. 18, fig 253). Let Ab′ and Ab be the elevation and plan of the line. From any point c′ in Ab′ draw VT at right angles to it meeting XY in T. From T draw TH at right angles to Ab produced meeting it at f. Then VTH are the traces of the required plane.

II. The problem is solved by imagining two semi cones, of proportionate size with their slant sides, forming the necessary angles to the two planes of projection and their axes meeting at the same point on XY. The required plane is tangential to the two cones, that is, its traces will touch the bases of the cones and pass through their vertices To limit the dimensions of the two cones their sides should be tangential to a sphere, the centre of which is the point on XY in which their axes meet.

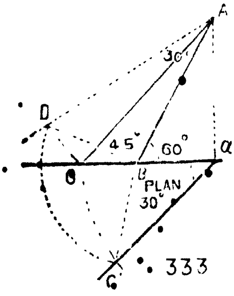
Select *a* any point in XY in which the axes of the cones meet and draw *cab'* through *a* perpendicular to XY. With *a* as centre and with any convenient radius *ad* draw a circle. Draw two lines *b'f* and *b'g* on two sides of *ab'* inclined at 15° with XY and touching the circumference of quarter sphere *ad* and meeting *ab'* in *b'*. Then *b'gt* is the small vertical cone showing the inclination of the plane with the H P Draw its semi base *gfk* on the horizontal plane. Draw another two lines *ce* and *ch* on the horizontal plane on two sides of *ca* inclined at 60° with XY and touching the circumference of the quarter sphere and meeting *ac* in *c*. Then *che* is the semi horizontal cone showing the inclination of the plane with the VP Draw its semi base *hme* on the vertical plane Draw from *b* and *c* two lines touching the circumferences of the bases of the cones and meeting XY in T. Then VTH are the traces of the plane

103. To determine the projections of a straight line which shall contain a given point and make given angles with the planes of projection. Fig 332.



Let the given point be P represented by its projections *p'* and *p*. Suppose the line is inclined at 45° with the HP and 30° with the VP From *p'* draw a line *p'b* at 45° with XY and *p'c* at 30° with *p'b*; from *b* draw *bc* perpendicular to *p'c*, and from *p'* draw *p'd* perpendicular to XY. Then *p'c* is the length of the elevation, and *bd*, the length of the plan, of the line passing through P and making the given angles with the coördinate planes With *p′* as centre and *p′c* as radius draw an arc intersecting XY in *a′*. Join *p′a′* which is the elevation. With *p* as centre and radius equal to *db* describe an arc; project from a′ to intersect the arc in *a*. Join *pa* which is the plan.

104 To determine the horizontal projection of a given angle θ when the lines containing it make angles of α and β respectively with the horizontal plane. Fig 333



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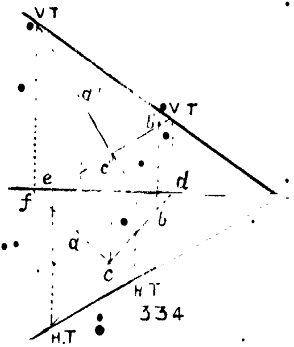


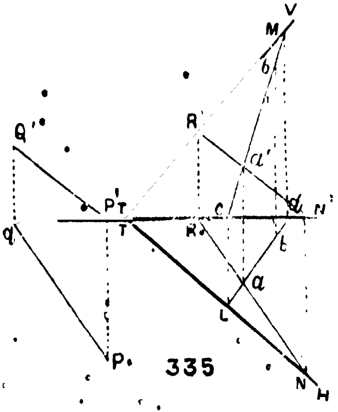
334

Let the given angle BAD = θ = 30°. Let AB be inclined at 60° with the HP and AD at 45° with the HP. Place the triangle BAD on the VP with the point B on XY. Then aB is the plan of AB. From the point A draw AC at 45° with XY and with A as centre and AC as radius draw arc CD intersecting AD in D. Join DB. Now revolve the triangle ABD on AB till AD touches the ground to bring it into its proper position. The length of plan of AD or AC is *a*C. With *a* as centre and *a*C as radius draw an arc and with B as centre and BD as radius draw another arc intersecting the first arc in C′. Join BC′. Then BaC′ is the horizontal projection of the angle BAD.

105. Determine the traces of a plane containing two given intersecting lines, AC and BC. Fig. 334.

Let a′c′, c′b′ be the elevation and ac, cb the plan of the intersecting lines AC, BC intersecting in C. Find the HT of AC and BC and VT of AC and BC.. Join the two H T's and the two V.T's they will meet the ground line in the same point, and they are the traces of the plane containing the two given lines AC and BC.

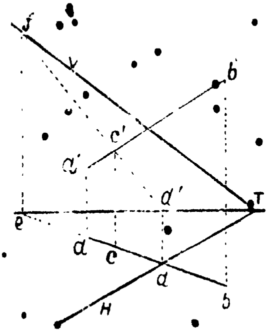
106 Given the projections of two lines which are not in the same plane, to determine the traces of a plane which shall contain one of these lines and be parallel to the other. Fig. 335.



335

Let AB and PQ be the given lines. If a line be drawn parallel to PQ from any point of AB and a plane be drawn containing this line and AB then it will contain AB and be parallel to PQ Find the traces L and M of the line AB Through any point *a* in *ab*, the plan of AB draw a line NaR parallel to pq the plan of PQ. Through *a*′, the projection of *a* on the elevation *a b*′ draw R′a′N′ parallel to P′Q′ the elevation. Find R′ and N the traces of the line RN drawn parallel to PQ. Join MR′ and NL and produce them to meet XY in T. Then NTM are the traces of the required plane.

107. To find the points of intersection of a line and a plane. Fig 336



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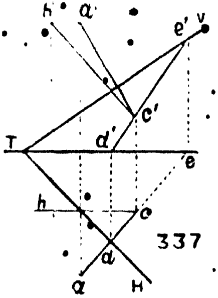


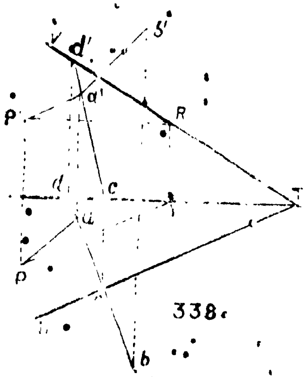
337

Let *ab* and *a′b′* be the plan and elevation of the given line and VT, HT the traces of the given plane. Imagine a vertical plane *feb* containing the line AB intersect the given plane. Its H.T. will be *eb* coinciding with the plan of the line AB and its V.T. is *ef* perpendicular to XY. Find the vertical projection *fd′* of common section of *feb* and the given plane. The point *c′* where *fd′* and *a′b′* intersect is the vertical projection of the point required. Project *c* on *ab* from *c′* which is the plan of the point

108. Determine the projections of a line, which shall contain a given point and be perpendicular to a given plane, and to find its true length. Fig 337

Let *a* and *a′* be the projections of the given point A and HTV the traces of the given plane From *a* and *a′* draw *ac* and *a′c′* at right angles to HT and VT. Produce *ac* to meet XY in *e*; draw *ee′* perpendicular to XY to meet VT in *e′*. From *d* where *ac* intersects HT draw *dd′* perpendicular to XY. Join *d′e′* intersecting *a′c′* in *c′*. Project *c* on *ac* from *c′* Then *a′c′* and *ac* are the projections of the line from A perpendicular to HTV. Draw *ch* parallel to XY and equal to *ca*. Draw *a′h′* parallel to XY and project *h′* from *h* Join *c′h′*. Then *c′h′* is the true length of the line which is now parallel to the VP.

109. From a given point to draw a perpendicular to a given line. Fig. 338.



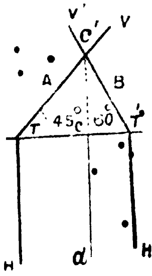
338.

Let p' and p be the given point and $a'b'$ and ab, the given line. The problem is solved by drawing a plane containing the point P and perpendicular to AB and then to find the projections of the line joining P with the point where AB intersects the plane. The traces of a plane perpendicular to AB are perpendicular to the plan and elevation of AB. Through p draw pr at right angles to ab meeting XY in r. From p' draw p′R parallel to XY. Draw rR perpendicular to XY meeting pR in R. Through R draw VT perpendicular to $a'b'$ the elevation of the line and from T in XY draw TH perpendicular to ab the plan. Then VTH contains P and is perpendicular to AB. Now it is required to find out the point where the line AB intersects the plane VTH. Let TH meet ab in c. Draw Cc perpendicular to XY. Let ab produced meet XY in d. Draw dd' perpendicular to XY meeting VT in d'. Join $d'c$ intersecting $a'b'$ in a'. Project a from a'. Then a' and a are the elevation and plan of the point where AB meets the plane VTH. Join $p'a'$ and pa which are plan and elevation of the perpendicular from the point P to the given line.

110 Project the intersection of two planes A and B inclined to the HP at 45° and 60° respectively and perpendicular to the VP. Fig. 339.

Let V″T′H′ and VTH be the traces of the planes B and A. As they are only inclined to the HP their vertical traces will make the given angles with XY and their horizontal traces will be at right angles to XY. Let V″T′ and VT intersect in c′. Project c on XY from c′, draw cd perpendicular to XY which is the plan of the line of intersection of the two planes, and the point c′ is its elevation.

111 Project the intersection of two planes A and B inclined at 60° and 30° to the HP respectively and their horizontal edges parallel to the ground line. Fig. 340.



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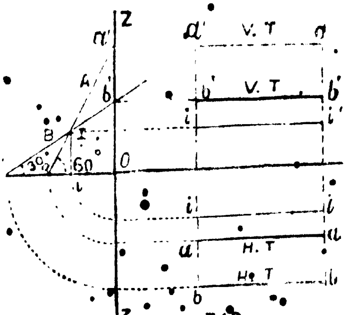
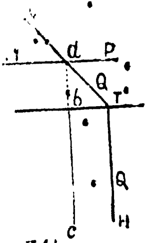


340

As the edges of the planes are horizontal and parallel to XY the planes are inclined in front and their line of intersection is parallel to XY. Draw *aa′* inclined at 60° and *bb′* inclined at 30° with the HP intersecting in I. These are the side views of the two planes A and B. Draw through *a′* a vertical line ZOZ′ intersecting *bb′* in b′ and the ground line in O representing the side plane. From O as centre and with *oa*, *ob* and *oi* as radii draw arcs intersecting oZ′ in three points. Draw from a′, b′ and the two lower points in oZ′ lines parallel to XY for the horizontal and vertical traces of the two intersecting planes. Draw from I and the point where oi′ transferred meets oZ lines parallel to XY for the projections of the intersection of the two planes.

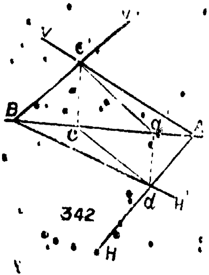
112. Project the intersection of two planes one inclined to the HP and the other parallel to the HP and 3" above it. Fig. 341.



341

Draw VTH the traces of the plane inclined to the HP only and draw a line parallel to XY at a height of 3" from it intersecting VT in *a* for the VT of the intersecting plane. As it is parallel to HP it has no horizontal trace. Project *b* in XY from *a*. Draw *bc* parallel to TH for the plan of the horizontal trace of the line of intersection ; the elevation is the point *a*.

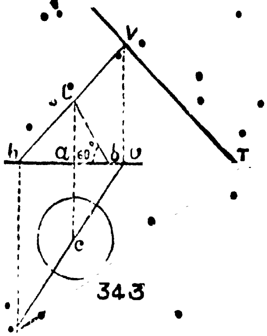
113. Project the intersection of two oblique planes. Fig. 342.



342

V,'BH' and VAH are the traces of two oblique planes Let *c'* be the point where the two vertical traces intersect and *d* the point where the horizontal traces intersect Then c′ and d are the traces of the line of intersection. Find c′d′ and cd the projection of the line (prob. 10 fig. 223).

114 Determine the traces of a plane which shall be inclined at a given angle *a* with the HP and shall contain a given straight line HV. Fig. 343.



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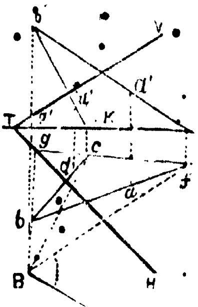


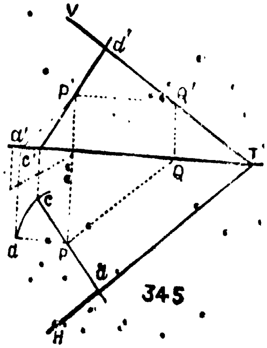
344

Let hV and Hv be the elevation and plan of the line and let the plane be inclined at 60° with the HP Find H and V the horizontal and vertical traces of the line. The traces of the required plane will pass through these points, Now imagine a cone with the slant side inclined at 60° with the HP placed on the horizontal plane, its vertex at any point C on the line HV. The required plane will be tangential to the cone and its horizontal trace must touch the circular base of the cone. From C a point in hV draw a line Cb at 60° with XY and Ca perpendicular to it. With *c* (plan in Hv) as centre and *ab* as radius draw a circle. From H the horizontal trace draw a line, tangential to this circle, meeting XY in T and join TV. Then TV, TH are the traces of the required plane.

115 Determine the angle which a line inclined to both planes of projection will make with a given plane. Fig. 344

Let HTV be the traces of the plane and *ab*, *a'b'* the plan and elevation of the given line From any point B in the given line AB draw a line BD, shown by *bd* and *b'd'* in plan and elevation, perpendicular to the plane HTV and find the angle between BD and BA The angle which BA makes with the given plane is the compliment of the angle which BD makes with BA. Draw *bd* and *b'd'* perpendiculars to HT and VT and find *c* and *f* the horizontal traces of the lines BD and BA. From *b* draw *bg* perpendicular to *fc* produced. Then $\angle cbf$ is the plan of the angle between BD and BA. To find the true angle the angle cBf is to be rabated on the HP on the line *fc*. The plan of the altitude of the triangle cBf is *gb* and its elevation is *g'b'*. Take *g'k* equal to *gb* and join *kb'*. Then *kb'* is the length of the altitude of the triangle. Produce *gb* to B making *g*B equal to *kb'*. Join Bc and Bf. Then cBf is the true angle between BD and BA. Draw Br perpendicular to Bc, then $\angle f$Br, the complement of $\angle cBf$ is the angle which BA makes with the given plane VTH.

116 To find the traces of a plane which shall contain a given point and make given angles with the planes of projection. Fig 345

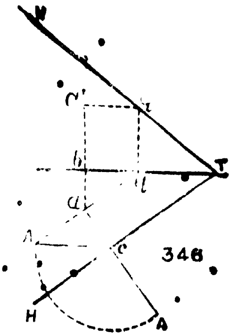


345

Let p and p' be the given point and 45° and 60° be the required angles of the plane with the HP and VP respectively. Through the point P a line is to be drawn inclined at (90-45) with the H P and at (90-60)° with the VP. Let *pc* and *p'c'* be the plan and elevation of this line. The question now is to draw the traces of a plane which shall contain the point P and be perpendicular to the line P C (see construction of prop. 109 fig 338). From p draw p Q at right angles to *cp*, draw QQ' perpendicular to XY and draw p' Q' parallel to XY meeting QQ' in Q'. From Q' draw V Q' T perpendicular to *c'p'* produced. From T draw

TH perpendicular to *cp* produced. Then VTH are the traces of the plane.

117. Find the rabatement of a given point A on the HP which lies in a given oblique plane V T H. Fig. 346.



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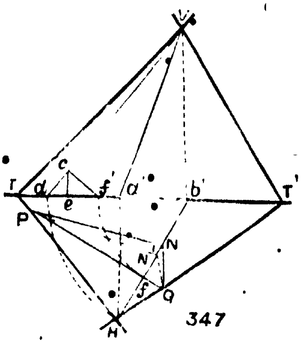


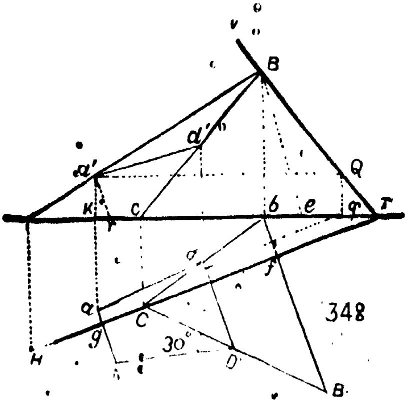
347

Let VTH be the traces of the given oblique plane and A a point in it. The projections of this point are *a'* and *a* when the plane is inclined to the two co-ordinate planes. From *a* draw *ac* perpendicular to HT and draw *a'b* perpendicular to XY At *a* on *ac* draw aA perpendicular to *ac* and join Ac. Then Ac is the distance of the point A in the plane from its horizontal trace. Produce *ac* to A' making CA' = CA. Then A' is thrown on the HP.

118. Determine the angle between two planes both of which are inclined to each of the co-ordinate planes but in opposite directions; their traces being given Fig. 347.

Let VTH and VT'H be the traces of the two planes. Find the projection V *a'* and H *b'* of the line of intersection between the two planes. Through any point *f* in H*b'* draw PQ at right angles to it terminating in P and Q on HT and HT'. From *b'* with radius b'f and b'H draw arcs meeting XY in *f'* and *d*. Join dV Then b'd V is the inclination of the line of intersection with the HP. Draw *f' c* perpendicular to dV. Make f N in H b' equal to *f'c* and join PN and N Q. Then P N Q is the angle between the two planes. If fN' be taken equal to f'e and P N' and N' Q be joined the angle P N' Q is the plan of P N Q.

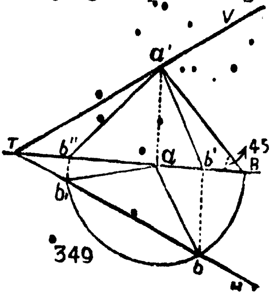
119. From a given point to draw a line to make a given angle with a given line. Fig. 348.



348

Let *a*, *a'* and B c, b C are the plan and elevation of the given point A and the given line BC. The line from A will make 30° with BC. Find the traces VTH of a plane containing the point A and the line BC or two given intersecting lines AB and BC (prob. 105, fig. 334). From *b* draw b fB' perpendicular to HT. Make *be* = bf and join B e. Make f B_1 = Be. From *a* draw a A' perpendicular to H T and make g A_1' = a'r (ag = Kr). Join B_1 with C and from A' draw A'D at 30° with C B_1'. The point A and the line BC is thrown on the ground as A' and B C. These are to be raised to their original position with the point D.' Draw D_1 d parallel to B'b meeting C b in d. Project d' in BC from *d*. Join *a'd'* and *a d*. These are the elevation and plan of the required line from A.

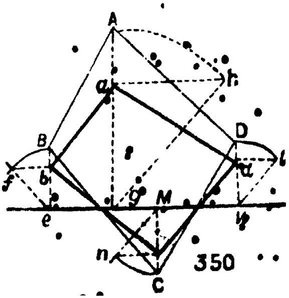
120. To project a line making an angle of 45° with the HP and contained by a given plane. Fig. 349.



349

Let VTH be the traces of the given plane. From any convenient point B in XY draw Ba′ at 45° with it meeting VT in a′. Draw a′ a perpendicular to XY. Now the line a′B is to be turned till its other end is on the HT. Then the whole line a′B will be in the plane. With a as centre and aB as radius draw an arc cutting HT in b and b_1 and Join *ab* and ab_1 then *ab* or ab_1 is the plan Project b′ and b″ on XY from b and b_1 and join *b′a′* or *b″a′* for the elevations.

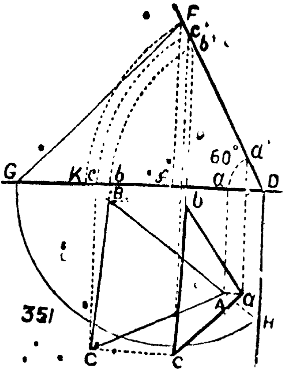
121. Rotate a given quadrilateral figure on an axis till it forms an angle of 45° with the plane of the original figure. Fig. 350.



350

Let ABCD be the quadrilateral, lying on the HP and it rotates on *e K* as axis It is required to find the plan of ABCD when it has rotated 45° on the axis *e K*. From the four points ABCD draw perpendiculars Ag, Be, CM, and Dk on the line *e K* and from the points g, e, M and K draw *gh*, *ef*, *Mn* and *Kl* at 45° with the perpendiculars and equal to them From *h*, *f*, *n* and *l* draw *ha*, *fb*, *nc* and *ld* at right angles to Ag Be, CM, and DK. Join ab, bc, cd and *da* Then *a b c d* is the plan of ABCD.

122. Determine the projection of an equilateral triangle ABC of 1″ sides its surface to be inclined to the H P at an angle of 60° and one of its edges at an angle of 45°. Fig 351.



351

From D any point in XY draw DF at 60° with it. This is the VT of the plane which will contain the equilateral triangle. Draw DH perpendicular to XY which is the H T of the plane. From any convenient point F in DF draw FG at 45° with XY. Draw Ff perpendicular to XY With *f* as centre and *f*G as radius draw an arc intersecting D H in H. With D as centre and DF as radius draw an arc cutting XY in K Join H K On H K draw ABC an equilateral triangle of 1″ side. Project the points *c*, *b*, and *a* on XY from C, B and A. From D as centre and with D c, D b and D a as radii draw arcs intersecting DF in *c′ b′* and *a′*. From B C and A draw lines parallel to XY and project the points *c b* and *a* on these lines from the points *c′ b′* and *a′*. Join *a b c*. Then *abc* is the required projection.

CHAPTER V.

INTERPENETRATION OF SOLIDS.

When two solids intersect each other they are said to interpenetrate, and the lines formed by the intersection of their surfaces, are called lines of interpenetration. The nature of the lines of interpenetration depends upon that of each of the surfaces. Thus if both surfaces consist of plane faces the lines of their intersection, will be straight lines; if one or both of the surfaces be curved the lines will consist of one or more curves.

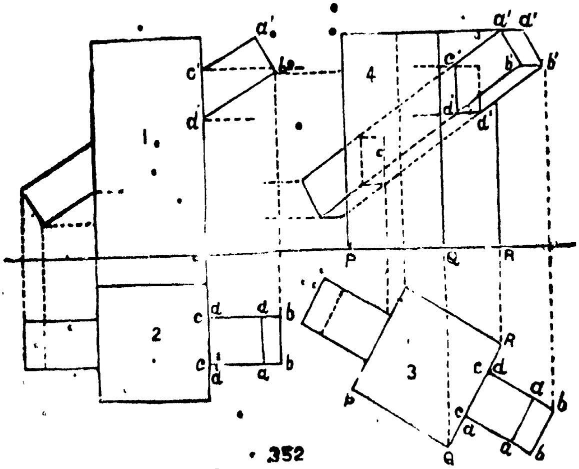
The principle involved is to determine points on the intersection of the surfaces of two solids by cutting planes. The intersected solid is supposed to be cut by a series of parallel planes which are first drawn either on plan or elevation of the solid, whichever is easier. The section surfaces give points which are common to the surfaces of the two solids penetrating each other. They are so chosen that the projections of the sectional planes shall always be straight lines in one, and straight lines, circles or other regular curves in the other projection. The series of points where these projections intersect are determined and joined, and as they are common to the surfaces of two solids the required intersection is obtained. A greater number of sections would confuse the points in determining the curve. To obtain the best results, comparatively few section surfaces should be carefully and judiciously chosen.

In almost all cases there are certain important points on the line of intersection whose projections should be found first, as the highest and the lowest points where the line disappears and reappears in view. We should first select those section surfaces which give us the important points. The correctness of the figure and easy solution depend on the careful selection of the section planes.

In the cases of interpenetration of prisms by other prisms the right line of intersection of surfaces are determined by finding two points in the right line. The two points may be determined by various methods.

Problems :—

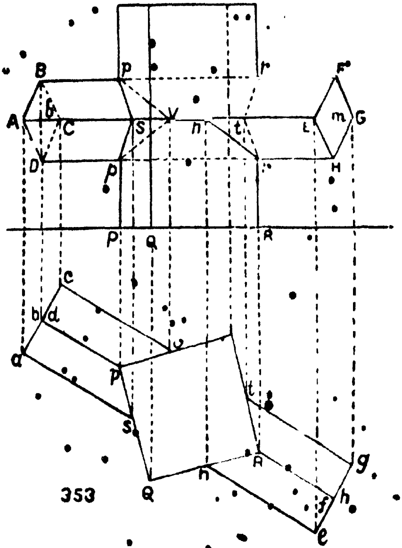
123. A rectangular prism penetrates another rectangular prism through the middle of a face at an angle with the H P. Find the lines of interpenetration when the arms are at an angle with the V.P. Fig. 352.



352

No. 1 is the elevation and No. 2 is the plan of a rectangular prism, intersected by another rectangular prism through the middle of a face. The penetrating prism is at an angle with the H. P. and is parallel to the V. P. No. 3 is a copy of No. 2 with the arms at an angle with the V. P. By projection from No. 3 and No. 1, No. 4 is obtained. First project the elevation of the vertical prism P Q R and then project the end ab of the arm as a' a' b' b'. The face cd ab (lower line) of No. 3 is in front and its line of penetration is obtained by projecting from the lower cd of No. 3 and from c' d' of No 1. The lower face d d b b of the arm is then seen and its elevation is obtained by projecting d d of plan No. 3 to intersect the horizontal from d' of elevation No. 1. In interpenetration problems the dotted lines need not be drawn.

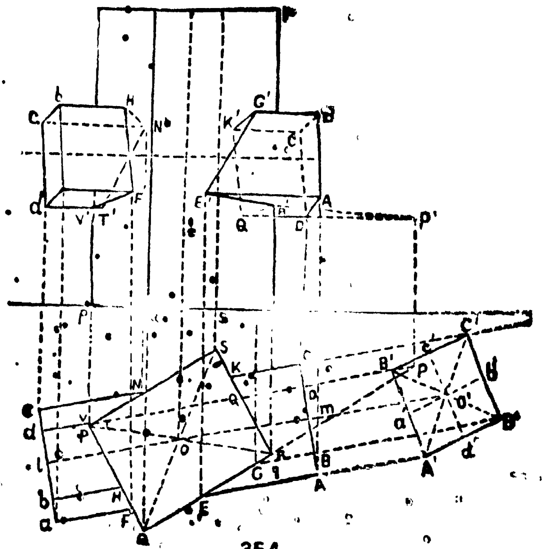
124. A rectangular prism, with two faces at 45° to the H. P. penetrating another rectangular prism through an edge. Draw the elevation when the two prisms are so turned that the arms are at an angle with the V. P. Fig 353.



353

P Q R is a square prism standing on the H. P. It is penetrated by a smaller prism a g e through the edges P and R. As this prism has two faces at 45° with the H. P. the edges bf and dh of the plan coincide and pass through the edges P and R of the square P Q R. First project the prism P Q R and then project the smaller prism for which proceed thus. Draw a horizontal line A C E G at about the middle height of the vertical prism. As the two faces A B F E and B C G F of the smaller prism are inclined at 45° to the H. P. the corners A C E G of the two planes are on this horizontal line. Project b d f h of the plan and where these two projections intersect the horizontal line ACEG of the elevation in b and m measure bB, bD and mF and mH each equal to ab or bc, and complete the ends. The line of penetration n'r and s p in the elevation are easily obtained by projecting n and R, s and P of the plan on the respective lines in the elevation. The lines of penetration t r and v p are behind and can not be seen.

125. **Determine the interpenetration of two rectangular prisms, one to be 1″×[illegible]″×1·9″ with its longer edges parallel to the V.P. and one of its faces inclined to the V.P.; the other prism to be 1·8″×6″×57″ with its longer edges parallel to the H.P and inclined 10° to the V P. No face of the smaller prism is parallel to the H P. The axes of the two prisms bisect each other. Fig 354.**



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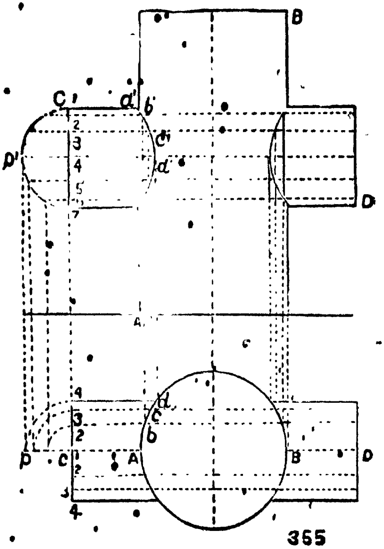
First draw the plan PQRS of the vertical prism in position by drawing the line PS at an angle with the V.P. Draw its elevation Pr. Draw diagonals of the plan for the plan of the axis, O. Through O draw a line at an angle of 10° with XY for the axis line of the horizontal prism. Measure on this line on the two sides of O, Ol and Om each equal to half the length of the horizontal prism and draw AC and ac perpendiculars to the axis through m and l. Now construct the supplementary end elevation of the horizontal prism using AC as the ground line. Produce lm the axis line to O′ marking mO′ = half the height of the vertical prism. Through O′ draw c′d′ inclined to AC the ground line, and a′b′ at right angles to c′d′ and make them equal to two sides of the horizontal prism. Complete the rectangle A′B′C′D′, the end elevation. Project A′B′C′D′ on AC to determine the edges of the horizontal prism on plan. Complete the plan of the horizontal prism.

The position of the edges of this prism in elevation are determined by making the heights of the points ABCD in elevation above XY equal to the distances of the points A′B′C′D′ from AC. Draw horizontal lines through these points till they meet the projectors from the corresponding points on plan on the left side *i.e.*, a, b, c, d.

For the line of interpenetration draw projectors from EGFH till they meet the edges Aa, Bb in elevation in E′,G′ F′,H′. The line of interpenetration E′G′ is in the face ABba. The next face is ADda which passes through two faces of the vertical prism, namely the faces QR and RS. Produce QR of the plan to meet the edge on dD of plan produced in P. Produce the edge dD of elevation till it meets a projector from P in P′. Join E′P′, cutting the edge Rr of the vertical prism in R′. Draw a projector from Q where the edge Dd of plan cuts the edge RS of the vertical prism till it meets the edge dD of elevation in Q′. Join Q′R′. Draw a projector from K where Cc of plan intersects RS of plan till it meets cC of elevation in K′. Join G′K′ and Q′K′. This completes the line of interpenetration for the right end of the horizontal prism. Proceed similarly with the left end. The lines of construction are shown in fig. 354. In the cases of prisms interpenetrating prisms, the method of obtaining common points on the surfaces of two solids by sectional planes is not applied as the lines of interpenetration are all straight lines which can be easily found by the direct projection of points in the lines.

126. A vertical cylinder, interpenetrated by another cylinder (1) their axes intersect at right angles (2) axis

of one is vertical and that of the other is inclined 45° to the H.P. Figs. 355 and 356.



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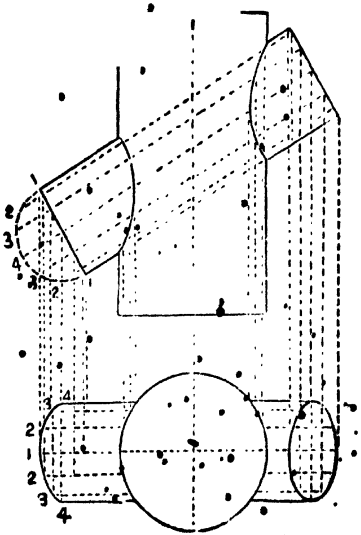
The principle of determining points on the intersection of the surfaces of two solids by cutting planes is necessary for all cases of interpenetration where the surfaces of one or both the solids are not plane faces. In the 1st case the sectional planes are horizontal and commences from the tangential plane to the horizontal cylinder. Sections are taken first in elevation and numbered as shown in fig. 355. The sectional planes cut the vertical cylinder in a uniform circle, and the horizontal cylinder in rectangles, except the first and the last which are straight lines. These sections are transferred to the plan by drawing a semicircle C'P'7 on 17 and producing the section lines to meet the arc of this semicircle. By projecting the points on the semi circumference to a horizontal line cp through the middle of the plan of the horizontal cylinder and transferring them on the line 44, the plan of the sections of the horizontal cylinder is determined. The first is the line CD. Then 22, 33, 44 are three other sections. The circle AB is the plan of all the sections of the vertical cylinder. The curve of interpenetration is determined by projectors from Abcd the points where the rectangles cut the circle till they intersect the corresponding lines of sections in the elevation. The lower part of this curve and the curve of interpenetration on the right side is similarly obtained.



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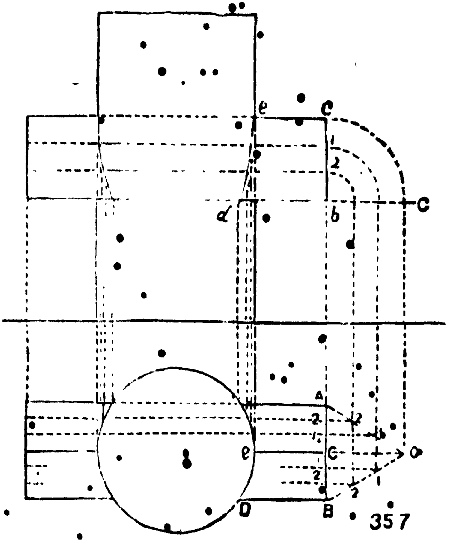
Case II. fig. 356. The sectional planes are parallel to the axis of the inclined cylinder and taken first in the elevation from which they are projected on the plan. The vertical cylinder is cut inclined but the plan of the sectional planes is the same as the circular plan of cylinder. The curve of interpenetration in elevation is determined by direct projection similar to case I.

127. A vertical cylinder interpenetrated by a triangular prism. Fig 357.



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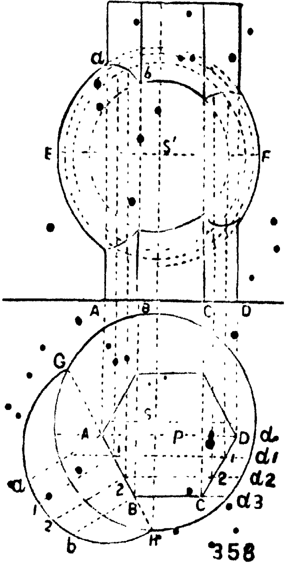
The triangular prism is placed with an edge on the top and a face at the bottom. The interpenetration of only one face in front is visible and obtained by taking two horizontal sections shown in plan and elevation No 1 and 2. To determine the ridge and section lines in plan draw ACB the triangular end on AB. Transfer the divisions on bc in elevation on a horizontal line bc' and then project on AC and BC Draw horizontal lines from 1 and 2 on AC and BC and from G on the plan of the prism. Draw projectors from the points where the horizontal lines from C, 1, 2 and B intersect the circle in plan till they meet the corresponding lines in the elevation. The sectional lines are shown on the upper side of the plan as the projectors coincide and for the convenience of projection. Join the points thus obtained by a fair curve for the lines of interpenetration in elevation.

128. A sphere, interpenetrated by a hexagonal prism; axis of the prism is vertical and is on one side of the sphere. Fig 358



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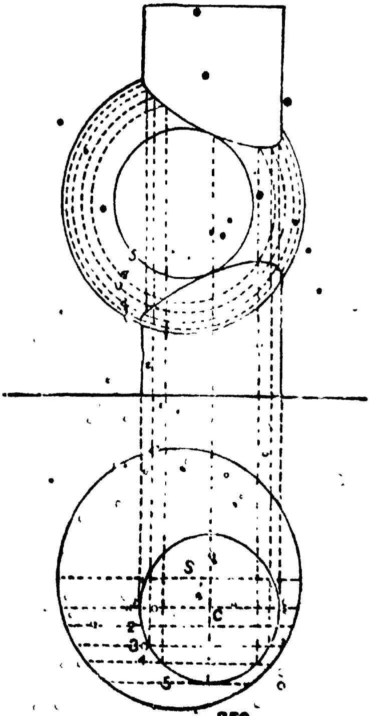
Take 4 vertical sections parallel to V P commencing from the centre P in the plan of the prism to the face BC. Transfer these sections on the elevation, where 4 circles, sections of sphere, intersect with 4 rectangles, sections of the prism. These are the points of the curve of interpenetration of the two side faces AB and CD of the prism with the sphere and 4 curves are obtained as ab etcetra in the elevation. The face BC is parallel to the V P so these curves in elevation are parts of a circle. As the point P is below S, the centre of the sphere, the section through AD cuts the sphere in a circle of diameter little less than the diameter of the sphere, therefore the edges A and D of the prism penetrate the sphere not in the circumference but little in the interior The elliptical curves of the sides may also be found by taking inclined sections of the sphere through the sides AB and CD. On GH the section line through AB draw a semicircle and from the 4 points in AB draw perpendiculars to AB to meet the semicircle. The heights of these perpendiculars are to be set off from the corresponding points in the diameter EF of the elevation, up and down for the points of the curve. The points will agree with those obtained by the first method.

129. A sphere, interpenetrated by a vertical cylinder axis of cylinder on one side of sphere. Fig. 359.



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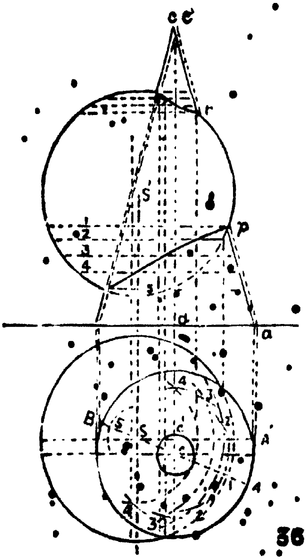
Let S be the centre of the sphere in plan and C the plan of the axis of the cylinder. Draw the elevations of the sphere and cylinder. Take 5 vertical sections, parallel to V.P. commencing from C in plan and obtain the elevations of the sections, 5 circles as sections of sphere in elevation and 4 rectangles and a line, for the prism. The last section is tangent to the cylinder, therefore its elevation is a line for the cylinder. Where the circles intersect the rectangles or the line, are the points for the upper and lower curves of interpenetration. Join these points. The cylinder penetrates the sphere beyond the circumference in elevation as C the plan of the axis of cylinder is below S the centre of the sphere in plan.

130. A sphere, interpenetrated by a vertical cone, axis of cone passing by one side of sphere. Fig. 360.



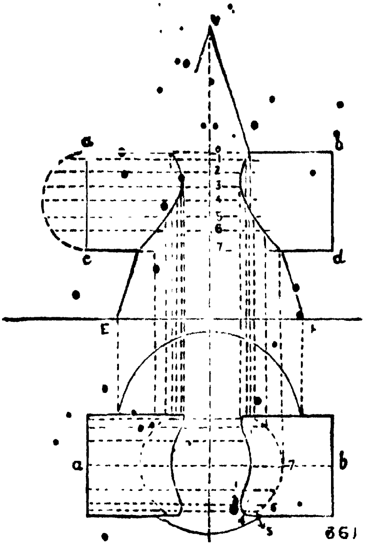
360

Let S and C be centres of the plans of the sphere and cone respectively. C is below S and on the right side. Join SC and produce both ways to meet the circumference of the base of the cone at B and A. The highest and lowest points of interpenetration lie on the vertical section passing through BA. Draw through S a line parallel to the ground line and with S as centre and SA as radius draw an arc AA′ cutting the horizontal line through S in A′. Similarly transfer the centre C to C′ on the line SA′. Project from C′ and find the apex of the cone in this new position. Project from A′ to meet the ground line in *a* and join *a′* with *c′* the new vertex. Where this line *ac′* cuts the sphere are the highest and lowest points of the curves on one side. Similarly the highest and lowest points on the left side may be found by turning B to the line A′S produced.

Take horizontal sections of the solid commencing from the highest points and ending on the lowest points thus found in the elevation of the solid. Five sections are taken for the curve of the lower portion, and 4 sections, for the upper portion of the cone. In the plan the sections will be represented by circles both for sphere and cone. The 1st and 5th sections touch at 1 and 5 in the plan and circles from sections 2, 3 and 4 intersect at 2, 3, and 4. Project from 1, 2, 3 and 4 in the plan to 1, 2, 3 and 4 section lines in elevation for the curve of interpenetration in elevation. The back curve not required to be shown may be found by projecting from points 5, 4, 3 and 2 above the diameter line of the plan. The lower penetration is below the diameter and so cannot be seen in plan. The upper curve of penetration is similarly obtained and shown both in plan and elevation.

131. A vertical cone interpenetrated by a cylinder, axes at right angles but not intersecting. Fig. 361.

Draw plan and elevation of the solid. Take the axis of cylinder in plan below the centre of the circle, the plan of the sphere. VEF is the elevation of the cone and *abdc* is the elevation of the cylinder. Take horizontal sections starting from the line *ab* and ending on the line *cd* in elevation. In the plan the sections are circles for the cone, and the line *ab* and rectangles for the cylinder. Where the circles intersect the corresponding line or the rectangles are the points of the curve of interpenetration in plan. Where projections from these points meet the respective section lines in elevation, are the points for the curves in elevation. The construction is simple.



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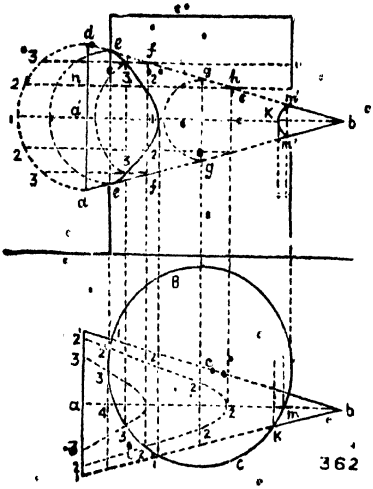
132. A vertical cylinder interpenetrated by a horizontal cone, axis of cone on one side of the cylinder. Fig. 362.

Draw cylinder and cone in plan and elevation. The axis *ab*, of the cone in plan is below C, the centre of the circle, the plan of the cylinder. Take horizontal sections 3, 2, 1, 2 and 3 from above the axis a′b′ to below it in elevation. The horizontal sections in plan is the circle ABC for all the sections of the cylinder and one triangle and two hyperbolas for the 3 sections of the cone. Where the triangle and the hyperbolas intersect the circle in plan are the points of interpenetration in plan and where projections from these points intersect the corresponding section lines in elevation are the points in elevation. To draw a hyperbola in plan. Take the case of No. 2 section. It cuts the cone in the line *nh*. Take 3 points in the line *nh* and draw verticals *ee*, *ff* and *gg* through them. Draw semicircles on *dd*, *ee*, *ff* and *gg*. The altitude of the points where the semicircles are cut by the section line 2*nh* from their respective diameters are to be set up and down on the projections from the corresponding points in *ab*, the axis of cone in plan. The points 2, 2, 2, thus obtained in



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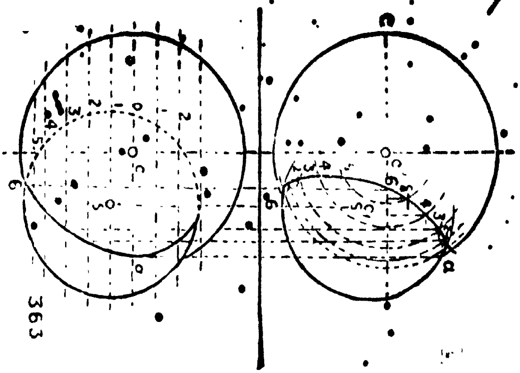
plan when joined is the curve of parabola. The apex of the parabola is found directly by projecting the point *h* in elevation on *ab* the axis in plan. Where this parabola intersects the circumference ABC are the two points of interpenetration which projected on the 2 section lines No. 2 in elevation gives the points in elevation.

133. A sphere interpenetrated by another sphere through one side. Fig. 363.

C and S are the centres in plan and C′ and S′, the centres, in elevation of the two spheres interpenetrating. Take vertical sections parallel to the V. P., 2, 1, 0, 1, 2, 3, 4 and 5, No. 0 passing through the centre C of the larger sphere in plan. In the elevation each section is a set of two circles intersecting for the two spheres. Find these intersections in elevation which are points common to the surfaces of the two spheres. Project these points to their corresponding lines in plan. The curve of interpenetration in plan is obtained.



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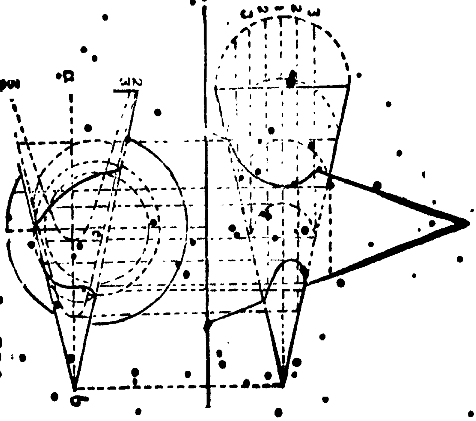
134. A vertical right cone, interpenetrated by another horizontal right cone, axis at right angles but not intersecting. Fig. 364.



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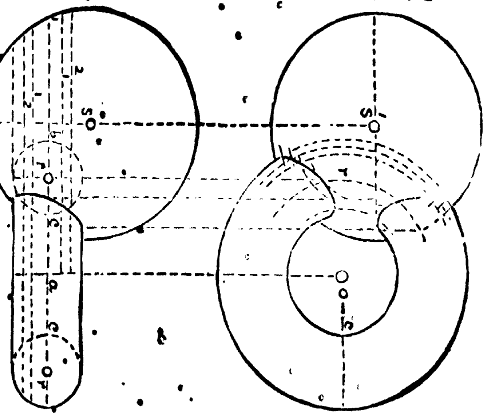
Draw plan and elevation of the two cones. In plan C is the centre of the circle, the plan of the vertical cone, and *ab* the axis of the horizontal cone, below C. Take horizontal sections in elevation Nos 3, 2, 1, 2, 3. In plan the sections for the vertical cone are circles and for the horizontal cone, a triangle and two hyperbolas. Where the triangle and the hyperbolas intersect the corresponding circles (sections) are the points of interpenetration in plan. Project these points to their corresponding lines (section) in elevation for the points of interpenetration in elevation.

135. A sphere interpenetrated by a ring. Fig. 365.



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S and S' plan and elevation of spheres. The ring passes by one side of the sphere. Draw a horizontal line below S', the centre of sphere in elevation and take o in it as centre of the ring. *cb* is the thickness of the ring. With o as centre and radii *oc* and *ob* draw circles for the elevation of the ring. Draw a line *bb* parallel to the ground line and below S, the centre of sphere in plan. On *bb* take *r* and *r'* as the centres of the thickness of the ring and complete the plan of the ring. Take vertical sections parallel to V. P. and draw section lines in plan. In elevation the sections will be represented by circles for the sphere and plane circular rings *i.e.*, two concentric circles for each section for the ring. Where the circles for the sphere intersect the concentric pair of circles for the ring are the points of interpenetration in elevation. These are to be projected on the corresponding lines in plan for points of the curve in plan.

CHAPTER VI

CAST SHADOWS AND THE PROJECTION OF THE LINE OF SEPARATION OF LIGHT AND SHADE.

If a surface, (as HP or VP or both or any other surface) receive light from any source, a portion of the surface is deprived of light, by the intervention of an opaque body. The part of the surface deprived of light is the shadow cast by the opaque body on the surface, or the cast shadow of the object. The opaque object which casts the shadow receives light from the source on one portion of its surface, the other portion being in shade. The line on the surface which divides the two portions is called the line of separation of light and shade; this line is evidently the locus of the point of contact of those rays which touch the surface; it is evident that these bounding or extreme rays are those which define the outline of the shadow cast on any surface The actual shadow is indefined and of varying intensity for reflected lights and other causes. We shall find out shadows with defined boundaries by assuming only one source of illumination, *i. e*, the sun, so far away and consequently so small that it may be treated as a point, and practically the rays are all parallel to one another. These shadows may be termed Geometrical shadows in distinction to the actual shadows seen in nature.

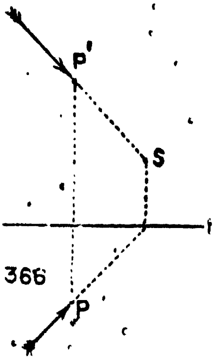
The method of obtaining the projections of Geometrical shadows cast by any object in plan and elevation depends entirely on the principle of finding the traces of lines or the intersection of lines with planes. The direction of light is fixed by means of the plan and elevation of any ray and is generally that in which they make angles of 45° with the ground line both in plan and elevation *i. e.*, the rays are supposed to be inclined at about 32° with each of the co-ordinate planes and are always drawn from the left.

Rule:—Imagine projections of parallel rays to pass through the prominent points of the projections of the object and find the traces of the lines of the rays with the vertical and horizontal planes. The lines joining the traces thus found will be the boundary lines of the shadow required. As the rays are parallel, lines parallel to a plane throw shadow on the plane parallel to themselves and on a plane at right angles to it, at an angle of 45° with the line of intersection of the two planes as the ray is taken inclined 45° to that line.

We shall deal with two kinds of shadows (I) those cast from a source of light at an infinite distance as the sun, when the rays are practically parallel to one another (II) those cast by a luminous point near at hand as a lamp when the rays are divergent.

PROBLEMS OF CASE I.

136. Shadow of a point in space on ground or wall Figs. 366 and 367.



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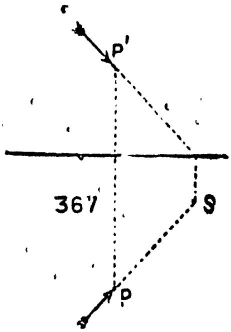

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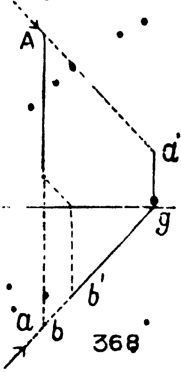
In fig. 366 P and P′ are the plan and elevation of P. Through P and P′ draw lines inclined 45° with the ground line. These lines represent the plan and elevation of the ray intercepted by the point P. Where the plan of the ray meets XY draw a perpendicular to meet the elevation of the ray in S. Then S is the shadow of P on wall. In fig. 367, the elevation of the ray meets the ground line first and the point S′ is the horizontal trace of the ray which is the shadow on the ground.

137. Shadow of a vertical line partly on ground and partly on wall. Fig. 368.

AB is the vertical line, its plan is the point *ab*. Through *a* draw a line inclined 45° with XY and through A and B in elevation draw two lines inclined 45° with the ground line. Find the traces of the rays through A and B. The trace of the ray through B is *b′* on the ground and the trace of the ray A is *a′* on the wall. Part of the shadow *b′g* is on the ground and is inclined 45° with XY and part of the shadow *ga′* is on the wall at right angles with XY. From this it is seen that a vertical line throws shadow on the wall parallel to itself, and on the ground inclined 45° with XY.



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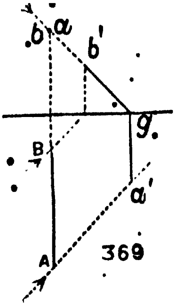

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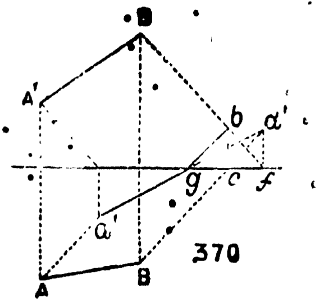
138. Shadow of a horizontal line at right angles to the wall, partly on ground and partly on wall. Fig. 369.

AB is the plan of the line perpendicular to the wall *i. e.* the VP and *ba* is a point in the VP for its elevation. Draw lines inclined 45° to XY from the two ends B and A in plan and from the point *ba*, and find the traces of the two rays A and B as a′ in the HP and b′ in the VP. The shadow of the line is a′gb′, the part a′g on the ground is parallel to AB and the part gb′ which is on the wall is inclined 45° with XY. This shows that the shadow of a line is inclined 45° with XY on the plane to which it is perpendicular. **Practically it may be noticed that shadows of vertical posts is vertical on the wall and is at an angle with XY on the ground.**

139. Shadow of a line inclined to both the planes of projection. Fig. 370.



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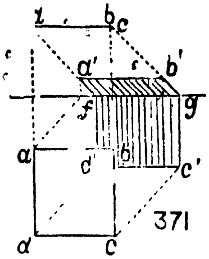

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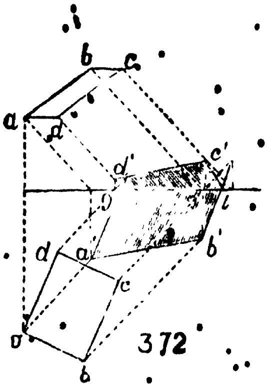
AB is the plan and A′B′ is the elevation of the line. The traces of the rays through A and B are *a′* in the horizontal plane and *b′* in the vertical plane. The question is how to join the two points *a′* and *b′* in two different planes The direction of the shadow on the horizontal plane can be obtained by finding the horizontal trace of the ray B. Let the ray in the VP from B′ meet XY in *f* and from *f* draw *fd′* at right angles to XY to meet BC, the plan of the ray, produced in *d′* Then *d′* is the HT of the ray through B; *a′* is HT of the ray through A. Join *a′d′* which would be the shadow if it was not intercepted by the wall. *a′d′* meet XY, the plan of wall, in *g*. Join *gb′* which is the shadow on the wall, and *a′g* is the shadow on the ground.

140 Shadow of a square lamina parallel to the ground and one edge parallel to the wall. Fig. 371.

The plan of the square piece is *abcd*, a square, and its elevation is *ad bc* a line parallel to XY. Find the traces of the rays through *a*, *b*, *c*, *d*, the 4 corners of the square in plan and elevation. As the edges *ad* and *bc* are at right angles to the VP their shadows on the ground are perpendicular to XY. The edges *ab* and *dc* are parallel to both the VP and the HP and the shadows of these two edges are parallel to XY. The shadow of the lamina is *fgc′d′* on the HP and *fgb′a′* on the VP.

141. The Shadow of a square lamina inclined to both the planes of projection. Fig. 372.



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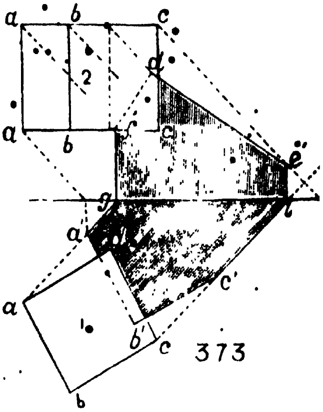

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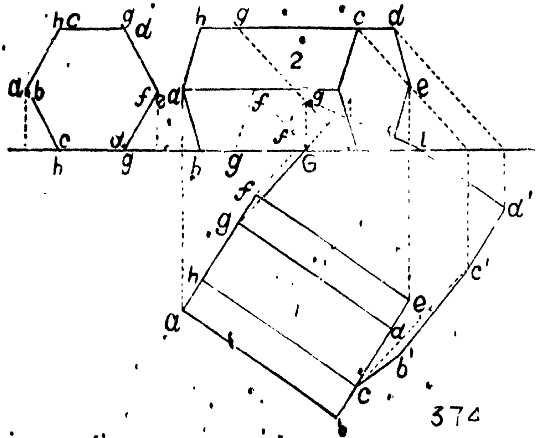
The plan of the square piece is *abcd* and its elevation is *adcb*. Find the traces of the rays through *a*, *b*, *c*, *d*, the 4 corners of the square in plan and elevation. The edge *ab* throws shadow on the HP and the edge *dc* on the VP. The shadows of the edges *bc* and *ad* are partly on the HP and partly on the VP. This is obtained by Prob. 139, fig. 370. The shadow of the lamina is a'b'lg on the HP and glc'd' on the VP.

142. The shadow of a cubical block with one face parallel to the ground and one face inclined to the wall. Fig. 373.

The plan of the cube is No. 1, a square, and its elevation is No. 2, two rectangles aa bb and bb cc. This is the first case of shadows of solid figures. The shadow is a plane figure and it is evident that some corners of solid cannot throw shadow as the rays from them are intercepted by the solid. In this case the top corner b and the bottom corner d cannot throw any shadow. The vertical edges aa and cc throw shadow partly on ground and partly on wall as a'gf' and c'le'. The shadow on ground is a'b' c'lg and on wall is le'd'f'g. Parts of the shadow are hidden by the plan and elevation of the object.

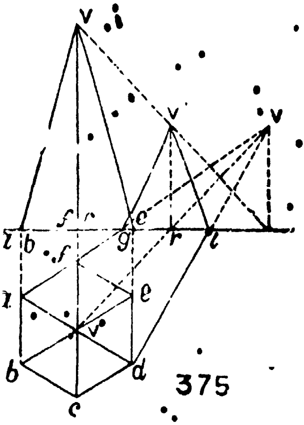
143. Shadow of a hexagonal prism lying on one face on the ground and its axis is inclined to the wall. Fig. 374.



The plan of the prism is *ae* No. 1. and the elevation is hd No. 2. The prism rests on the HP on plane hcdg which cannot throw shadow. Rays from corners a, top h and top g are intercepted by the solid and cannot throw shadow. The shadow on the ground is cb'c'd'LGg and on the VP is Lg'f'G. The corners b, top c and top d appear on the right, the edge dg is partly on the ground and partly on the VP. The corners top g and f appear within the elevation and fg comes out as f'Gg partly on VP and partly on ground;

144. Shadow of a hexagonal pyramid resting on its base on the ground. Fig. 375.

The plan of the pyramid is abcdef and its elevation is aev. As the pyramid rests on its base on the ground its shadow is the shadow thrown by its two edges fv and dv. The points f and d are on the ground. It is now required to find the shadow of the apex v. The trace of the ray passing through V is V' in



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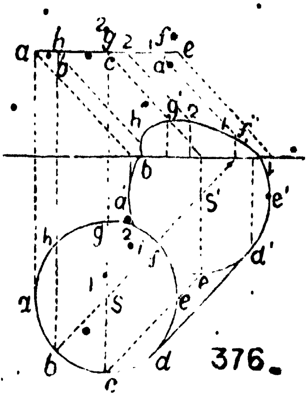


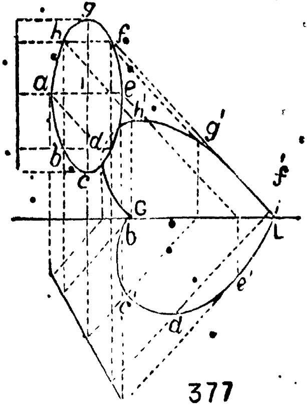
376.

the VP. This V' cannot be joined directly to f and d which are on the HP. Find the trace of the ray v on the HP as V'' and join fv'' and dv'' cutting the ground line in g and l. Join g and l with v'. Then the shadow of the pyramid is fgldef on the ground and glv' on the wall.

146. Shadow of a circular disc parallel to the ground. Fig. 376.

The plan of the disc is a circle *abcdefgh*, No. 1 and the elevation is a line *ae*, No. 2. As the disc is parallel to the ground the portion of the shadow that is thrown on the ground is part of a circle ba'c'd'e'L and the part thrown on the wall is part of an ellipse Lf'2g'h'b. Find s' the trace of the ray through s the centre of the circle and with s' as centre and with the radius of the disc as radius draw a circle till it is intercepted by the ground line at bL. The points h'g'21f' of the ellipse are found by traces of rays through hg21f. Join the curve passing through these points with b' and L on the ground line for completion of the shadow.

146. Shadow of a circular disc perpendicular to the ground but at an angle with the vertical plane. Fig. 377.



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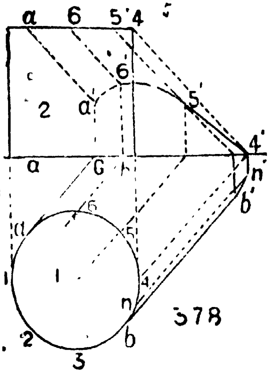


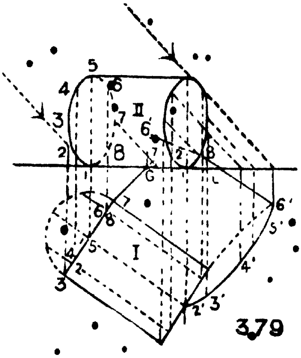
378

The plan is a line ae with the same divisions as No. 2 of fig. 376. The elevation No. 1 is obtained from the plan and a vertical line ge equal in length with ae the plan and with the same divisions. The shadow is elliptical both in plan and elevation. The traces of the rays which are on the ground are first found as b′c′d′e′ and the traces of the rays which are on the VP are a′h′g′ and f′. The curve through the points b′ c′ d′ e′ can be continued by finding the trace of the ray through f on the ground plane as f″, where the curve e′ f″ cuts the ground line is the point L to be joined by a smooth curve to f′. Similarly the point G on the left side is found

147. Shadow of a vertical cylinder resting on ground. Fig. 378.

No. 1 is the plan and No. 2 is the elevation of the cylinder. As the cylinder is resting on the ground there is no shadow of the bottom plane. The two tangent lines, aa b5 throw shadow as aGa′ and bb′ and the right semicircle of the top plane a654b throw shadow as a′6′5′4′b′. To obtain the proper curve between 4′ and b′, a point n is taken between 4 and b and its trace is found in n′.

148. Shadow of a horizontal cylinder inclined to the wall. Fig. 379.

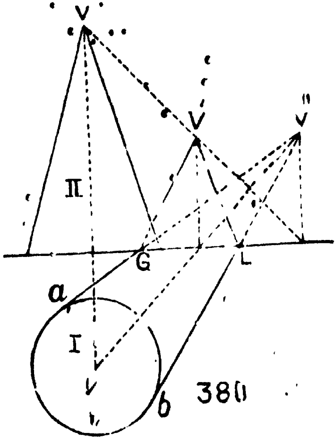


379

No. I is the plan and No. II is the elevation of the cylinder. The cylinder rests on the ground on line 11. As the rays are inclined 45° and comes from the left the portion of cylinder from 22 to 66 which receives light will throw shadow. The shadow line 22 almost coincides with line 11. On the right side the points 2, 3, 4, 5, and 6 will throw shadow and a curve 2′ 3′ 4′ 5′ 6′ is obtained by joining the traces. The line 66 will throw shadow on the ground parallel to itself Draw 6′ L parallel to the length of the cylinder. Find the trace of the ray from the left point 6 on the wall which is 6′ on the left. Join L with 6′ on the VP. The traces of the rays through 3 4 5 on the left fall inside the shadow. The trace of the ray through left point 8 falls on the ground. Find the trace of the ray through the left point 7 on the ground. Join the point 8′ with 7′ by a fair curve; it cuts the ground line at G. Join G with the left point 6′ on VP by a fair curve which completes the shadow.

149. Shadow of a cone resting on its base on the ground. Fig. 380

I and II are the plan and elevation of the cone. As the base of the cone is on the ground it throws no shadow. First find the trace of the ray which passes through V the vertex on the ground plane in V′. From the point V″ draw two tangent lines on the circumference of the plan of the cone as V′a and V″b. V′ab would be the shadow of the cone if there was no VP



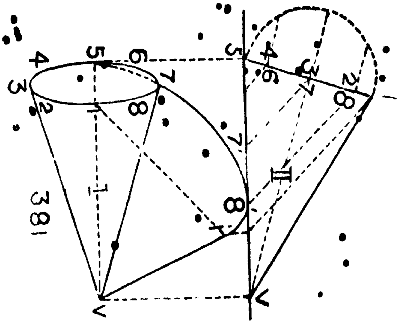
380

intervening. Find the trace of the ray through V on the VP which is V′. Join V′ with G and L where the shadow on the ground meets the wall. Then the shadow on ground is aGLb and on the wall is GV′L.

160. Shadow of a cone lying on its slant face on the ground with its axis parallel to the V. P. Fig. 381.

I and II are the plan and elevation of the cone. The cone lies on the ground on the line 5 V. Find the 8 points on the rim of the base of cone in plan and elevation by drawing a semicircle on 15, in elevation. The trace of ray through the point 1 is 1′ in plan. Join V. the vertex in plan with 1′. The tarces of rays through 2, 3, 4, fall within the shadow. The point 5 is on the ground. The shadow of the point 6, 7 and 8 is 6′, 7′, and 8′. Join the points 1′, 8′, 7′, 6′, and 5 by a fair curve. The full shadow is V 5 6′ 7′ 8′ 1′ V. Part of it is covered by the plan which is dotted.

151. Shadow thrown by 4 steps, the top being a landing. Fig. 382.



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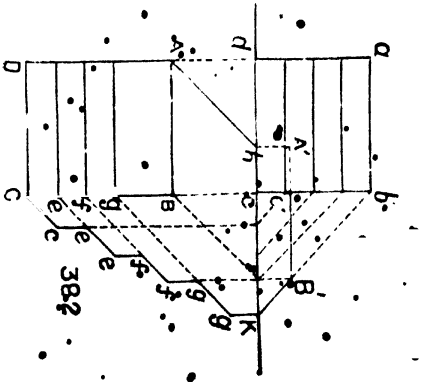
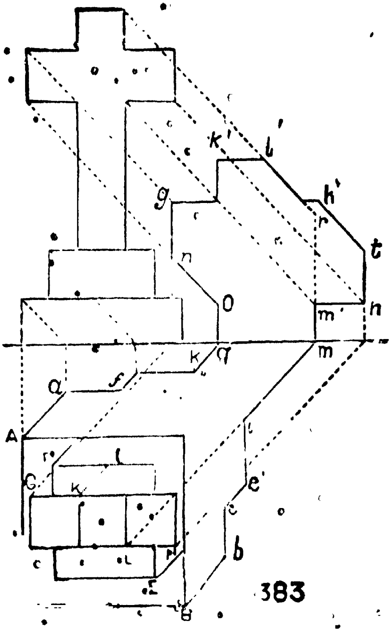


382

- The steps are shown in plan as ABCD and in elevation as dcba. The heights of the four corners at c, e, f and g are shown by lines inclined 45° to *xy* as *cc*, *ee*, *ff*, and *gg*. The shadow of edge gB of the landing is partly on ground and partly on the wall as gkB'. The height at A is *da* and its shadow is AhA'. Part of the shadow on the wall is covered by the elevation and is shown dotted.

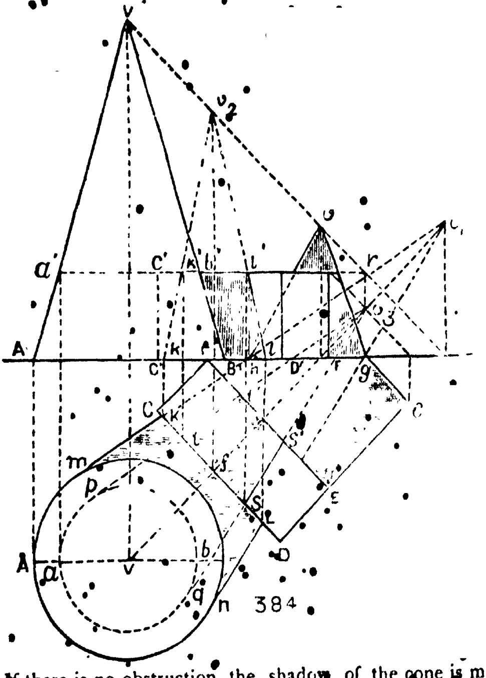
152. Shadow of a cross standing on a square slab upon another slab. Fig. 383.



383

Hints for drawing the shadow.—In plan AB is the lower and FE the upper slab. KL is the shaft of the cross and GH is the arm of the cross of the same section as the shaft. First find the shadow of the lower slab which is AafcbB. The shadow of the upper slab comes out of it at f and c as fkle'c. The shadow of the shaft comes out of the last at k and l as kqml on ground plane and qnm'rl'k'oq on the wall. A little portion near r is covered by the shadow of the arm. The shadow of the arm is gh'thm'ong. The projection of the arm from the shaft should be little more than the height of the portion above the arm.

163 Shadow of a cone first falls on the ground, is then obstructed by a rectangular prism (lying on one face on the ground and at an angle with the wall), and then appears on the wall. Fig. 384.

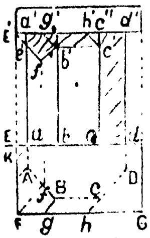


384

If there is no obstruction the shadow of the cone is mngh on the ground plane and hgv on the wall (prob. 149 fig. 380). The prism of wood CE interferes, and the shadow rises up on the nearest vertical face of the prism. Project KL on the elevation as kl. Find v^3, the vertical trace of the ray, through the vertex V on the prolongation of the vertical plane of the prism by assuming CD as the ground line. By joining kl with v^3 the shadow kl l'k' on the vertical face of the prism is obtained.

The shadow then appears on the top face of the prism seen in plan, by bending from k'l'. Project k'l' on CD at *t* and *v*. Produce the top plane of the prism indefinitely and imagine the cone to be cut at a'b' in level with it. Find v' the horizontal trace of the ray through v at level a'b'. From v' draw tangents to apq, the horizontal section of the cone in level with the top of the prism. The shadow on the top of the prism is *tss't'*. The prism throws shadow E*eg*F. Part of the shadow of the cone on the wall (hgv) is hidden by the prism.

154. The shadow of a square cap on an octagonal prism. Fig. 385.



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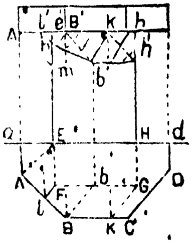


386

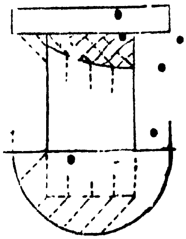
The shadow is to be shown on the vertical faces of the octagonal pillar shown in plan as aABCD*d*. The capital in plan is EFG. Here the ground line is not XY, but the outer line of the plan of the prism aABCD*d*. From the corners aABC of the plan draw lines inclined 45° back to intersect the plan of the capital. Project these points on the lower line of the cap in elevation. From the corner F of the cap draw a line inclined 45° to meet the ground line at *f*. From the points E'*a'g'h'* on the lower line of the cap in elevation draw lines inclined 45° and intersect them respectively by projections from the corresponding points *a*ABC of the ground line. The points *e'f'b'c'* of the shadow are obtained. The plane CD of the pillar is in shade in elevation. The shadow is *a'e'f'b'c'* which merges in the plane of shade c''d.

The following shadows are all projected in the same way. The figures explain the constructions.

155. Shadow of an octagonal cap on a square prism. Fig. 386.

156. Shadow of a circular cap on a square prism. Fig. 387.

157. Shadow of a square cap on a cylinder. Fig. 388.



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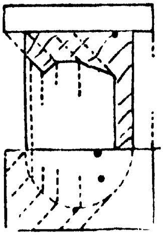

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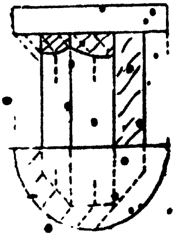
158. Shadow of a circular cap on an octagonal prism. Fig. 389.

159. Shadow of an octagonal cap on a cylinder. Fig 390.

160. Shadow of a circular cap on a cylinder. Fig. 391.



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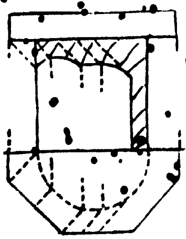

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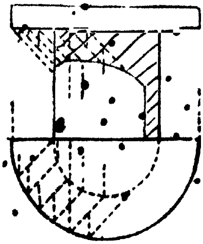
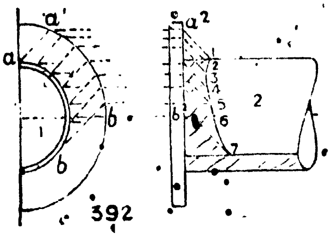


391

161. Shadow of a circular flange upon a horizontal cylinder. Fig. 392

In fig. 392 the side elevation 1 serves the same purpose as the plans in the preceding examples. *aa'* is the first and *bb'* the last tangential ray throwing shadow. Take any number of points between *a'* and *b'*, and draw lines inclined 45° from those points to the root of the flange. Transfer the points in the flange of the side elevation to the flange in the front elevation fig 2 from a^1 to b^2 Draw lines from the points a^2 to b^2 inclined 45°. and intersect them by horizontal lines from the points *a* to *b* on the cylinder Points 1, 2, 3, 4, 5, 6 and 7 of the shadow are thus obtained. From the tangent point *b* draw a horizontal line for the shade portion of the horizontal cylinder



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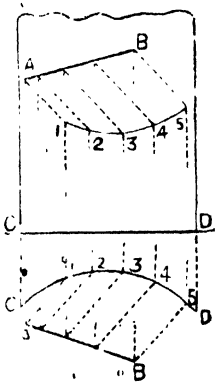


393

A few cases of shadows thrown on curved walls, or on the corner of two walls meeting at a point, are now shown.

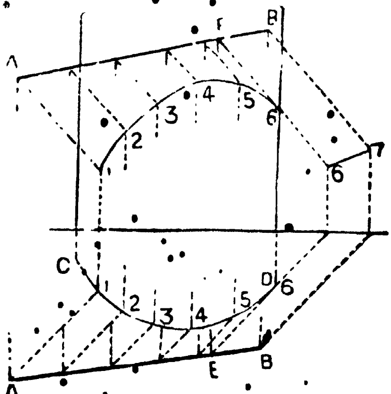
In these cases the construction is the same as in previous cases, the only difference is that the ground line is changed.

162. To determine the shadow of a straight line cast on the concave face of a curved wall. Fig. 393.

AB and A'B' are the plan and elevation of a thick wire throwing shadow. CD is the concave face of a curved wall. The ground line is C D The shadow 12345 in elevation is obtained by drawing lines inclined 45° from the points in-

A'B', the elevation, and intersecting them by projection from the corresponding points in the ground line. The *concavity* of the wall in elevation is expressed by the bending *down* of the curve of the shadow.

163. Shadow of a straight line on the convex face of a curved wall. Fig 394.



394

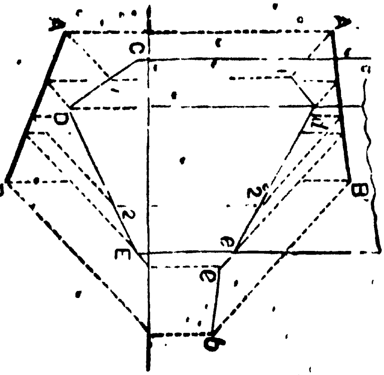
AB and A'B' are the plan and elevation of a thick wire. CD is the convex face of a curved wall. The part AE of the wire throws shadow on the curved wall and the remaining portion EB throws shadow on the original vertical plane. Here the *convexity* of the wall CD is expressed in the elevation by the bending *up* of the curve of the shadow.

164. Shadow of a straight line cast on the outside corner of two vertical walls meeting at an angle. Fig. 395.

A B and A'B' plan and elevation of the wire throwing shadow. The outside corner of a wall in plan is CDE. Part of the shadow *eb* falls on the vertical plane beyond the wall. The shadow on the walls is *1d 2e* bending up on the outside corner.

165. Shadow of a straight line cast on the inside corner of two vertical walls meeting at an angle. Fig. 396.

Here the shadow falls on the inside corner of two walls and the shadow bends down. The construction requires no explanation.



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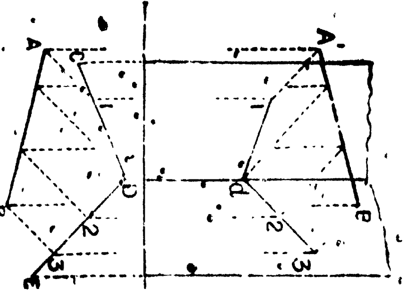
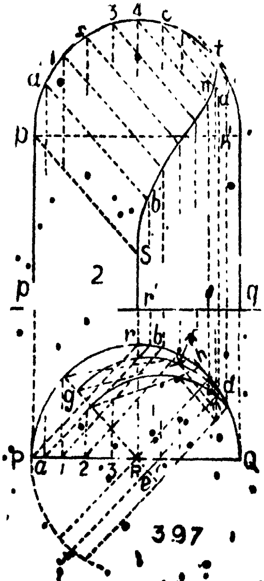


396

166 Project the shadow cast on the interior of a semicircular niche 1½ inches wide and 2½ inches high, the top of the recess being round and the lower part flat. Fig. 397.

Let 1 and 2 be the plan and elevation of the niche or recess.

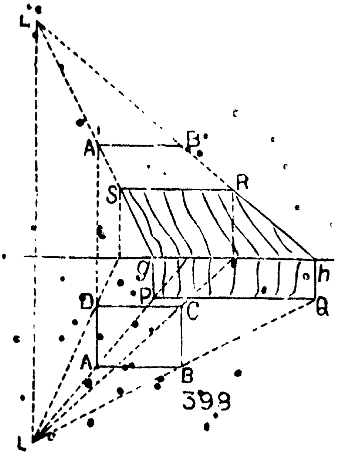
Take any number of points a', c', etc. in the arch over the niche and find the corresponding points a, c, etc. on PQ in the plan. Draw Pr in plan inclined 45°, the portion of the curve of the back of the recess from p to r is in shade. From p′ in elevation draw p′s inclined 45° and intersect it by projection from r. Then pp′sr′ is the shadow for the cylindrical portion of the niche. The shadow of the edge of the arch begins from s. Draw the line *ab* inclined 45° and draw the projector bb′ till it intersects the line a′b′ inclined 45° from a′ in elevation. The other points of the shadow below the soffit 4 of the arch are obtained similarly.



397

Draw the line *cd* in plan with 45° set square and the projector dd′ till it meets the line c′d′ inclined 45° from c′ in elevation. Now the curve of the soffit of recess in a plane on the line *cd* is to be constructed, which can be done in plan by producing the line dc till it meets the continuation of the plan of the recess in f. Bisect df in *e* and with *e* as centre construct a semicircle on df. The quarter of the arc is shown as *dg*. Draw dh perpendicular to fd and equal to *h' d'*, the height of the point d' from the horizontal line through the springing of the arch at p'. Join hg in plan, where it cuts the arc *gd* is the point m Draw the line mn perpendicular to *fd*. Draw the projector nn' to meet the line c'd' inclined 45° from c' in elevation. Then n' is a point of the shadow. Find other points in the same way and join them to complete the shadow. The shadow terminates at *t* the tangential point of the arch at 45°.

167. Shadow of a rectangular lamina cast from a luminous point. Fig. 398.



398

The principle of construction for finding the points of shadows in these cases differs from the previous cases in this that the rays are drawn radiating from the projections of the luminous point to the corners of the object instead of parallel rays through them inclined 45° to the ground line.

Let L and L' be the plan and elevation of the luminous point and ABCD is the plan of the rectangular lamina and A'B', its elevation.

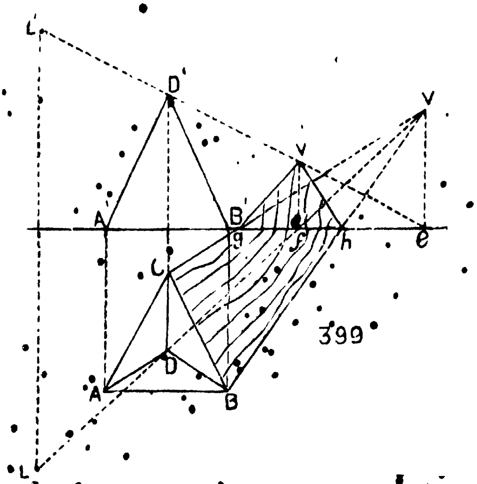
Draw lines from L through the corners A, B, C and D of the lamina till they meet the ground line.

Draw lines from L' in elevation through A' and B' the two corners of the elevation of the lamina.

Find the traces of the rays as in previous cases for the points of the shadow.

The shadow of the edge AB is PQ on the ground plane and of the edge DC is SR on the vertical plane. The portions of the shadows of AD and BC on the ground are parallel to the edges. Draw Pg and Qh parallel to AD and BC. Join gS and hR for the shadow on the V. P.

168. Shadow of a triangular pyramid cast from a luminous point. Fig. 399.



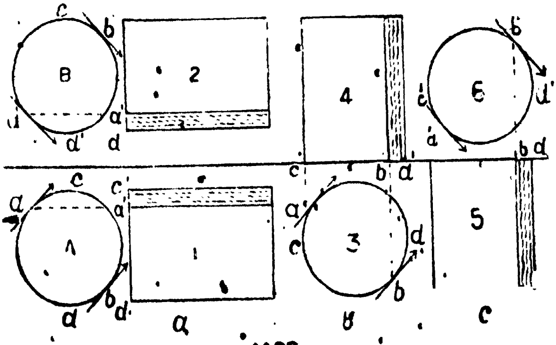
399

L and L' are the projections of the luminous point. AB CD is the plan of the pyramid and A'D'B' is the elevation.

Here it is required to find the shadow of the vertex D on the ground plane and on the VP. Join L with D in plan and produce it and join L' with D' in elevation and produce it. V and V' are the traces of the ray on the ground plane and vertical plane respectively. Join C and B in plan with V meeting the ground line at g' and h. Join g and h with V$'$. The shadow on the ground plane is ChBg and in the VP, ghV$'$.

In these cases as the light is near the object the shadow is much bigger.

169. Determine the line of separation of light and shade in plan and elevation of a given cylinder which is (*a*) parallel to both the planes of projection, (*b*) perpendicular to the ground, (*c*) perpendicular to the wall. Fig. (400a, b, c).



400

(*a*) 1 and 2 are plan and elevation of the cylinder when it is parallel to both the planes of projection. Here the cylinder is horizontal.

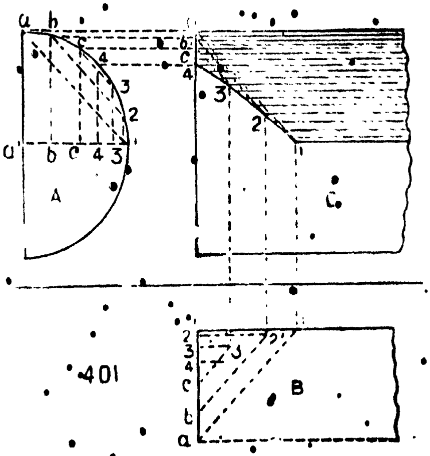
Let A be the side view of the cylinder in plan No. 1. Draw the tangential rays inclined 45° to A. The portion adb receives light. Of the semicircle cad which is in front the portion ac is in shade. Draw a projector from a. The upper portion of the cylinder is in shade.

Let B be the side view of No. 2, the elevation. Here the lower portion of the cylinder is in shade.

(*b*) When the cylinder is perpendicular to the ground the plan is a circle. Draw tangential rays to this circle. The right hand portion bd is in shade.

(*c*) When it is perpendicular to the wall the elevation is a circle. Draw tangential rays to this circle in elevation. Draw projector from b. The right hand portion bd of the plan is in shade.

170. Determine the line of separation of light and shade in elevation, of a hollow semi-cylinder placed horizontally with the concave side in front. Fig. 401.



401

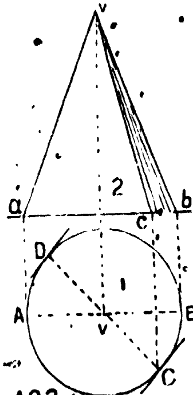
A is the side elevation of the end of the hollow semi-cylinder. B is the plan and C is the elevation.

Take points in the rim of the upper half of the side view as a,b,c, 4,3,2,1. Draw a' 1 the semi-diameter of the side view and draw perpendiculars from the points a, b &c. on this line.

In B the side a1 is equal to a' 1 of the elevation. Transfer the points in a 1 of the elevation to a1 of the plan. Draw rays inclined 45° from the points a,b,c of the rim in side elevation. The rays intersect the arc at 1,2,3 respectively. The ray through 4 is tangential. Draw lines inclined 45° from a,b,c of the plan B intersecting horizontals from 1,2,3 of the plan in 1'2'3.' Project the points a,b,c and 4 on the side line of the elevation representing the end of the semi-cylinder. Project horizontally from 1, 2, 3, of the rim in side elevation till they meet projectors from 1', 2' and 3' in plan. These are the 3 points of the curve of the shade. The 4th point ray is tangential therefore the curve ends at 4 in the elevation.

Draw a horizontal line from 1 in c for the lower line of the shade.

171. Determine the line of separation of light and shade of a given right cone. Fig. 402.



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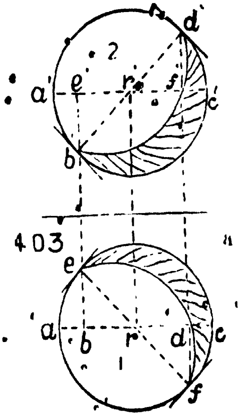


403

1 is the plan and 2 is the elevation of the cone. Draw tangential rays inclined 45° at D and C of the plan. The portion CB is in shade. Project the point C to c on the ground line. Join cV in elevation. The portion cVb in elevation is in shade

172. Determine the line of separation of light and shade of a given sphere Fig. 403.

Let 1 and 2 be the plan and elevation of the sphere. Draw rays tangential to the elevation at b' and d' and to the plan at e and f. In elevation the portion b'a'd' receives light therefore the 2 points in the line of separation of light and shade in elevation is b' and d'. Similarly the two points in the line of separation of light and shade in plan is e and f as eaf receives light. Project the points e and f on the elevation as e' and f' and project the points b' and d' in plan as b and d. By this method four points in the line of separation of light and shade are obtained in plan and elevation. An elliptical curve can fairly be drawn through these 4 points in each case. Only half the curve can be seen in each case which are drawn

THE END

CALCUTTA UNIVERSITY QUESTIONS.

L. E. AND B. E. FROM 1885.

1. Find by construction a point on a given line which shall be equidistant from any two points situated outside the line.

2. Construct a triangle whose base shall be 2", altitude 3" and vertical angle 30° without using a protractor.

3. Draw a six sided irregular figure and reduce it by the parallel ruler to a triangle of equal area.

4. A tin cylinder 6" diameter is penetrated, centrically and at right angles with the axis, by a smaller cylinder of 3" diameter; show how you would proceed to draw the outline of the whole on the sheet of tin before bending it into the form of the larger cylinder.

5. Draw a square threaded screw 2" outside diameter, thread ¼" thick and ¼" deep. About 3" of the length of the screw may be drawn.

6. Draw to scale a triangle with sides 3, 4 and 5 inches and without the aid of a protractor make a triangle equal to it in area and having one angle of 60°.

7. In an angle of a square of two inches side construct a square one-half the area of the original square.

8. After construction of the square within the square as required in question 7, the remaining portion of the larger square will be a double rectangle, draw by simple geometric construction a square equal to the double rectangle.

9. Draw a diagonal scale of yards, 1 yard to 1 inch, to show feet and inches.

10. A plane surface in plan is represented by a square of two inches side inclined to the horizontal plane at an angle of 30°. On the projection of this plane draw a line inclined to the horizontal plane at an angle of 20°.

11 Draw the horizontal and vertical projections of a right pyramid with a horizontal base of one inch side. Height of pyramid is 4 inches and the axis is inclined 30° to the horizontal plane, the horizontal projection of the axis being inclined at the same angle to the vertical plane.

12 Draw projections of the section of a vertical cone made by a plane inclined 45° to the horizontal plane showing also the form of the section in its own plane.

13. Draw the projections of a V threaded screw with an outside diameter of 2 inches, the pitch and depth of screw being $\frac{1}{4}$ of an inch.

14. Make an isometric drawing of a wooden cross made by the intersection of two beams 1 foot square and 4 feet in length. Scale $\frac{1}{12}$.

15. Assuming the horizontal and vertical traces of a plane, (neither of which are perpendicular to the intersection of the horizontal and vertical planes) and the projections of a point, draw a line from the point perpendicular to the plane and shewing the point of contact with the plane.

16. Construct a plain scale of miles shewing furlongs, $\frac{1}{63360}$ natural size.

17. Construct a diagonal scale of metres proportional to a scale of 1 yard = 1 inch reading decimetres and centimetres and make on it a length of 1 metre 2 dec. 3 cent.

1 metre = 39·37 inches.

18. Draw an equilateral triangle of 3 inches side and by simple geometrical construction make a square of equal area.

19. Construct a rectangle of 8 square inches area, and from any point in one side cut off by a straight line a portion equal in area to $\frac{1}{3}$ of it.

20. Draw the horizontal and vertical projection of a section of a right cone above its base made by a plane inclined 45° to the horizontal plane and whose horizontal trace is inclined at an angle to the vertical plane. Develop the section and show a practical method of drawing the correct figure.

21. Draw the horizontal and vertical projections of a right octagonal prism, one face being inclined 30° and 2 corresponding edges 20° to the horizontal plane.

22. Assuming the horizontal and vertical traces of a plane, not parallel to either plane, and the projections of a point above it draw any line inclined to the plane at an angle of 45° from the point and show the point of contact with the plane.

23. Assuming the horizontal and vertical projection of a sphere of 2 inches diameter, shew the point of contact of a plane tangential to it and inclined 45° to both vertical and horizontal planes. Draw the traces of the plane.

24. Given the trace and inclination of a plane to the horizontal plane; find its trace upon and inclination to the vertical plane. Construct the angle between the traces of the plane.

25. A right cone has its base inclined 15° to the horizontal plane. Draw the shadow thrown by it on that plane, the light

passing over your left shoulder so that the projections of a ray will make an angle of 45° with the horizontal and vertical planes.

26. Draw the isometric projection of a combined cube and tetrahedron, side of cube is 2 inches and face of tetrahedron the largest that can be inscribed in side of cube. Construct the isometric scale.

27. Construct without the aid of a protractor, a pentagon of 2 inches side.

28. Make by simple construction a square equal in area to three fifths of the pentagon in question No. 27.

29. Construct a simple scale of 1 mile 2 furlongs = 2 inches.

30. Construct a comparative diagonal scale, to one of 1 mile = 1 inch, of knots to show decimals of a knot to two places of decimals. 1 knot = 6080 feet.

31. Draw the horizontal and vertical projections of a line :—

(*a*) Inclined 30° to the horizontal plane, and in a plane parallel to the vertical plane.

(*b*) Inclined 30° to the horizontal plane and in a plane inclined 30° to the vertical plane.

(*c*) Inclined to the horizontal plane at an angle of 45° and in a plane inclined 45° to both horizontal and vertical planes.

32. Draw the vertical and horizontal projections of a tetrahedron whose base is inclined 40° to the horizontal plane, and the horizontal projection of one of the sides of whose base is inclined at 30° to the vertical plane.

33. Develop the surface of the above tetrahedron on the horizontal plane.

34. Explain and illustrate by diagram the principle of isometrical projection. Draw the isometrical plane.

35. On a plane whose traces are inclined to both horizontal and vertical planes, draw the projections of a line inclined at any given angle to the horizontal plane.

36. Assuming the projections of a point above a plane, the traces of which are inclined to both horizontal and vertical planes and which makes an angle of 30° with the horizontal plane, draw the projections of a line passing through the point and inclined at an angle of 45° to the given plane. Show the point of contact with the plane and of contact with horizontal or vertical plane or with both.

37. A right prism with octagonal ends has one of its faces inclined 30° to the horizontal plane, and the horizontal projection

of its axis makes an angle of 25° with the vertical plane. Draw its orthographic projections, the prism being 3 inches in length and a side of regular octagon is ¾ inch.

38. A plane inclined 45° to the vertical plane cuts the prism in position in question 37. Show the section on the horizontal and vertical planes.

39 Draw in isometrical projection a tower obtained by adding a right pyramid, one inch in height to one end of the prism in question 37, the base of which coincides with the end of the prism.

40. A sphere of 1 inch radius rests on the horizontal plane. Assuming the horizontal trace of a plane, not perpendicular to the vertical plane, draw a plane parallel to the assumed plane touching the sphere and indicate the point of contact.

41. Divide a triangle into two equal parts by a line parallel to one of its sides.

42. Draw a circle which shall touch the two lines of a given angle, and pass through a point within the angle.

43 Taking the horizontal and vertical projections of any two points so that the straight line joining them is not perpendicular or parallel to either plane, find the correct length of the line and its angle of inclination to one of the planes.

44 Draw the horizontal and vertical projections of a right prism, 4 inches in length with a regular pentagonal base of 1 inch side, resting on a point of its base on the horizontal plane, the axis being inclined to the horizontal plane at an angle of 30°, the horizontal projection of the axis being inclined at the same angle to the ground line.

45. A cylinder with a diameter of base of 1 inch interpenetrates the prism as projected under the conditions of question 44, so that the axis of the cylinder is in the same plane with and cuts the axis of the prism at right angles. Draw the elevation of the combination.

46. A sphere of 2 inches diameter raised one inch above the horizontal and 1 inch away from the vertical plane is intersected by a plane inclined 45° to the horizontal plane and inclined also to the vertical plane. Draw the correct section and show the point of contact with the sphere of a plane parallel to the cutting plane and touching the sphere.

47. Three lines, which are 1¼, 2, 2½ inches in length and are mutually at right angles, converge from above, and meet at a point in the ground plane; the 2½ inch line prolonged forms angles of 60° with the vertical and ground planes. Find the plan and elevation of these three lines.

48 A right hexagonal prism of 1½ inches side and 3 inches height stands balancing on one of its points, the plane of its axis forming an angle of 45° with the vertical plane; draw the plan and

elevation. Also draw the contours of the solid taken at every ½ inch level measured from the ground plane.

49. A survey of tract of country, 14 miles by 10 miles in extent is required to be plotted so as to fit on an "imperial" sized sheet of paper. Note what scale would you adopt. Imperial size = 30 in. × 22 in.

50. Draw the plan and elevation of an equilateral triangle, of 2 inches sides, one of them being at right angles to the vertical plane and one inch above the horizontal plane, and the other inclined at an angle of 30° to the latter.

51. Find the shadow in plan and elevation cast by a cube of 2 inches side, parallel with, and 3 inches distant from, the vertical plane, the cube being placed evenly upon a right square prism of one inch side and 3 inches height, standing diagonal wise upon the horizontal plane. Light at conventional angle.

52. Draw a rectangular figure with an area of 45 square inches and state to what scale it should be measured to give an area of 20 square feet, draw the scale.

53. Draw three hexagons of 1½ inch side in mutual contact and construct by graphic means a square of half their combined area.

54. A square disc of 1½″ side rests upon a plane whose vertical and horizontal traces make angles of 20° and 50° respectively with the ground line and lies in such a position that one of its points touches the ground plane and one of its diagonals is parallel with the horizontal trace. Draw the plan and elevation of the disc.

55. A thin plate 2″ × 1″ poised in the position indicated below is set in motion through space in the following manner :—

It first moves vertically to the extent of 2″, and is then tilted on its right edge through an angle of 45° and travels horizontally to the left for a distance of 2 inches. Supposing the space thus cut through to form a solid, draw its form and the shadow cast by it on the vertical and horizontal planes by light falling at the conventional angle.

56. A cone, whose height is 3 inches and base 1½ inches in diameter, is supposed to be lying on its side with its axis parallel to the vertical plane. It is intersected on the side towards the spectator by a plane, parallel to the vertical plane and inclined at an angle of 75° to the horizontal plane. The portion removed is such that a segment of the base of the cone is cut off whose versed sine is a third of the diameter. Draw the plan and elevation of the cone after intersection.

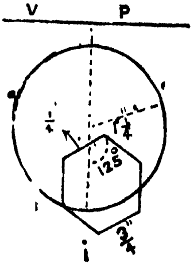
57. Project the shadows cast at the conventional angle of 45° on the vertical and horizontal planes by two square prisms (1″ sides and 3 inches height) one of which stands on end, square with the vertical plane, and one inch distant from it, and the other is laid flat on one of its sides, touching the former prism and with its axis

inclined to the vertical plane at angle of 75° in such a manner that the end near to that plane is turned to the right and is half an inch away from it.

58. What briefly are the object and advantages of the isometric method of projection? Explain the isometric scale.

59. Four equal spheres of 1½ inch diameter, each touching the other three and thus forming a pile, rest on the ground plane in such a way that one of the lines which join the centres of the three spheres in contact with the ground is at right angles to the vertical plane. Obtain the plan and elevation.

60. The given figure represents a right cone, whose height is 3", intersected by an upright hexagonal prism of ¾" sides. Supposing the prism to be removed, draw the elevation of the cone.



61. Construct a diagonal scale of 1½ inches to the mile to show miles, furlongs and chains and mark on it a length of 2 miles, 3 furlongs, 2 chains. What is the representative fraction of this scale?

62 In an equilateral triangle whose side is 3 inches draw three equal circles, each circle to touch the other two as well as two sides of the triangle.

63. A hexagonal prism 4" long, 1" side, stands with one edge of its base on the H. P. and has its axis inclined at 60° to the H. P. and 30° to the V. P. Draw its plan and elevation.

64. A cone 4" high with a base of 2 inches diameter is cut by a plane passing through the centre of its height and making an angle of 30° with the plane of its base; project the section on the plane of its base and determine the true form of the section.

65. A line 2 inches long has its lower end 1" above the H. P. and 2" from the V. P. it is inclined 45° to the H. P. and 30° to the V. P. Determine its horizontal and vertical traces.

66. Assume any two lines as the horizontal and vertical traces of a plane and show how to determine the true inclinations of the above, to the two planes of projection.

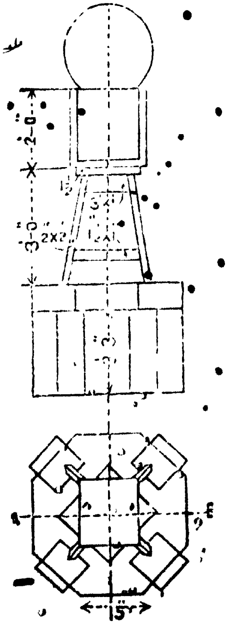
67. Find the vertical and horizontal traces of a plane tangential to a given sphere the point of contact to be any point in the lower half of the sphere.

68. A right cone whose base is 1½ inches in diameter and height 4 inches, rests upon the horizontal plane so that both its base and apex touch the horizontal and vertical planes. The cone is made to rotate on its axis in the direction away from the vertical plane and one complete revolution is effected. Draw the plan and elevation of the cone in its original and subsequent positions.

69. A glass inkstand in the form of a square pyramid 4" high has a base of 2½ inches sides, of which one makes an angle in plan of 30° with the ground line. At a height of 2½ inches above the horizontal plane the pyramid is divided so as to form a hinged lid to the inkstand. Draw its plan and elevation assuming the lid to be opened to an angle of 45° in the direction most divergent from the vertical plane.

70. Draw plan and elevation, to a scale of $\frac{1}{24}$, of the group of objects shown roughly in the marginal figures. Also draw the section on the line AB.

71. Two semicircular vaults intersect in CD. They are equally wide. Draw the intrados resulting from a section taken along the line EFG. ED is one fourth of CD



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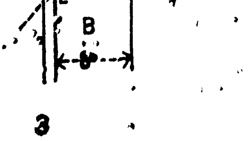
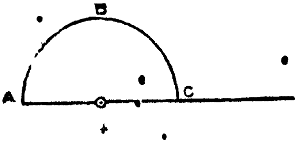


3

72. Three spheres of 1", 2", and 3" diameter, respectively rest on a horizontal plane, touching one another. Draw a full size plan, showing by dotted lines, the parts invisible from above and showing also their points of contact.

73. A cone of base BC dia. 2" and height 3" lies on the horizontal plane, with its axis parallel to the V. P. It is cut by a horizontal plane ¾" above the H. P. Draw plan of the parabolic section showing clearly by projection the method of obtaining it.

74. ABC is a semicircular lamina of radius 7 inches which rests vertically with its base AC in a horizontal plane. A string of 22 inches has one end fixed at C and is coiled round the arc so that the other extremity is at A. The free end A is revolved in the vertical plane of the lamina so as to keep the string always taut. Trace out the curve described by A until it reaches the horizontal plane again.



75. Take three points A, B, C in a straight line. AB = 1½ in, BC = 2¼ in. B between A and C. Draw a rectangle, the sides of which are in the ratio 2 : 3 vertex at B, and two sides passing respectively through A and C. Draw another rectangle of double the area circumscribing it and leaving equal spaces between, all round.

76. Draw in isometric the dial of a clock of 5" diameter, half full size with the Roman numbers. Draw the hour and minute hands showing 10 minutes passed 9 o'clock. The dial is horizontal.

77. Determine the shadow cast upon the interior of a hollow cylinder in section through axis by a circular piston fitted into it which is not in section. The cylinder is vertical.

78. Draw a sector of a circle 3" radius containing an angle of 120°. Bisect the two sides and the arc, and form a triangle by joining the three points. This sector is the development of the surface of a cone and the triangle, lines on its surface. Determine the plan and elevation of the cone with the lines when it rests with its base on the ground.

79. A cylindrical pipe turning at an angle of 120° is to be formed out of a sheet of metal measuring 4 ft. by 9½ inches, the arms of the pipe to be each a foot long. Draw the elevation of the pipe when it rests on the ground on one end with the other arm at 45° with the V.P. Draw also the development of the surface of one arm, scale ⅓.

80. Draw the plan of a portion 4 ft. square of a floor made up of hexagonal tiles of 5 inches sides. Scale $\frac{1}{12}$.

[illegible] 1 75 in. diameter touches both planes of projection. Determine the traces of a plane touching the sphere and inclined 60° and 45° to the horizontal and vertical planes respectively.

82. A cylindrical chimney 18″ diameter is to be fitted to a sloped roof of a smith's shop. The rise of the roof is ¼th of the span. Draw the hole in the roof for the chimney.

F. E. 1885 to 1910.

1. From a point near the extremity of a given line erect a perpendicular. Drop a perpendicular upon a given line from a point which is nearly over the extremity of the line.

2. Cut off the corners of a given square so as to form a regular octagon, without the use of a protractor.

3. Construct a rectangle one side of which is 1½″ and the diagonal 4″.

4. Draw the plan and elevation of a truncated hexagonal right pyramid with the following conditions. Base, a regular hexagon, each side of which is 1½″. Two sides of the base are inclined at an angle of 15° to the ground line. Height of pyramid 3″. The truncating plane is inclined at 45° with the horizontal and is perpendicular to the vertical plane, and cuts the pyramid at 2″ up from the base. One elevation is required showing the appearance of the solid as viewed in a direction parallel to the ground line and another of the solid as viewed at right angles to the ground line.

5. A hexagonal obelisk in the centre of a flight of steps, the bottom, middle and top steps are respectively 4″, 3″ and 2′ square and each ½′ high. The obelisk is a right hexagonal truncated pyramid, each side of the bottom is 1′ and each of the top is ½″. The height of the truncated portion is 5″ from the base. Draw isometrically.

6. From points in a straight line and without using a protractor set off angles of 45°, 30° and 20°.

7. Draw a square which shall have an area equal to the difference in area of two given squares.

8. Given a square of 3 inches side, construct a rectangle of equal area, one of its sides being [illegible] inches.

9. Draw the horizontal and vertical projections of a straight line 3 inches in length when

(1) it is parallel to both planes of projection;

(2) it is perpendicular to the horizontal and parallel to the vertical plane;

(3) it is inclined 45° to the horizontal plane and in a plane perpendicular to both horizontal and vertical planes;

(4) it is inclined 20° to the horizontal plane and in a plane inclined 30° to the horizontal and perpendicular to the vertical plane.

10. On a scale of .75 inch to a foot draw the horizontal and vertical projections of an octagonal column 5 feet high and 1 foot width of face, standing on a square pedestal of 8 feet side and 1 foot deep, one of the faces of the column being inclined 20° to the vertical plane.

11. A plane inclined 45° to the horizontal plane cuts the top of the column in question No. 10, show the section in both horizontal and vertical planes.

12. Make an isometric drawing of the column in question 10 and cut by the plane as in question 11.

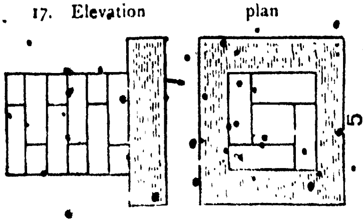
13. Draw a scale of 3 inches to a mile to read multiples of 10 yds.

14. In the middle of a square of 2 inches side construct geometrically another square which shall contain one-third of the area of the original square.

15. A square prism having a base of 1 inch side, and a height of 2 inches is made to balance on one of its points, and thus swung round till the plane of its axis makes an angle of 45° with the ground line. Draw its elevation in the position.

16. Two egg shaped sewers meet at an angle of 120°, draw their intersection.

17.



Draw the brick pillar detailed in the marginal fig. in isometric projection.

18. What do you understand by the terms:—
Rhomboid, Trapezium, Prism, Tetrahedron, Orthographic Projection, Development, Helix, Traces of planes.

19. A straight wire 2 inches in length is bent at its centre into a right angle, draw its projections on the horizontal and vertical planes when one arm is inclined 30° to the horizontal plane and the other is in any plane not parallel to the vertical plane.

20. Draw the projections of a cube standing on one of its angles one face being inclined 30° to the horizontal plane.

21. Project on the horizontal and vertical planes the figure of a square prism of 2 inches sides aud 4 inches length intersected by a right cylinder of 1 inch diameter, the axis of prism and cylinder intersecting at right angles.

22. What do you understand by the term Isometric Projection. Draw the Isometric Scale.

23. Draw the isometric projection of a box without a lid, shewing its interior, the invisible lines to be dotted.

24. Without the aid of a protractor construct on separate bases of 3 inches in length the following figures :—Square, Rhombus with angle of 60°, and Isosceles triangle with angle at base 75°.

25 Construct a square equal in area to the Isosceles triangle in question 24

26. Construct a rectangle of which two opposite sides should be 2½ inches in length and equal in area to a square of 2 inches side.

27 Make a scale of knot comparative to a scale of 1 miles=1 inch. 1 knot=1.15 miles.

28. Draw the horizontal and vertical projections of a line 1½ inches in length :—

(1) when it is parallel to the horizontal and perpendicular to the vertical plane ;

(2) when it is inclined at an angle of 45° to both planes.

(3) when it is inclined 45° to the vertical plane and its horizontal projection is inclined 35° to the trace of the two planes.

29. Draw the horizontal and vertical projections of a right pyramid with a hexagonal base of ¾ inch side and height of 3 inches resting on one of the sides of the base, the base being inclined 75° to the horizontal plane, and the horizontal projection of the axis inclined 30° to the vertical plane.

30. Develop the surface of a pyramid of the dimension given in question 29.

31. Draw the isometrical projection of a hexagonal column 4 inches in height and ¾ inch side of hexagon. The figure should be drawn to isometric scale which must be constructed.

32. The distance between Calcutta and Hooghly which is known to be 26 miles measures nine inches on a map. Construct a scale for this map to read miles, furlongs and chains.

33. Draw a pentagon of two inches side and reduce it geometrically to a square of equal area.

34. A triangle, the plan of which forms an equilateral figure, has its three points 10, 14 and 21 units above the horizontal plane and its longest side which measures 20 units parallel with the vertical plane. Draw the vertical projection and find the real form of this triangle, assuming each unit of measurement equivalent to [illegible]/8″.

35. Why are scales used by Engineers and Architects? What do you understand by the terms "diagonal scale", "vernier scale", and "isometric scale"? By what expressions are the scales $\frac{1}{18}$, $\frac{1}{48}$, [illegible], $\frac{1}{1200}$ ordinarily alluded to?

36. In the middle of a square of 3 inches side draw a parallel square of 4½ square inches area.

37. Draw the plan of a portion 4 feet square of a floor made up of hexagonal tiles of 4 inches sides. Scale one inch = one foot.

38. Set up an elevation of the tiling referred to in question 37, supposing the floor to be sloping towards you at an angle of 30°.

39. Draw a diagonal scale of [illegible] to read feet and inches.

For what purposes are the following scales ordinarily used:—

(*a*) Half an inch to the foot.
(*b*) One sixteenth of an inch to the foot.
(*c*) Fifty feet to the inch.
(*d*) Eight miles to the inch.

Give their representative fractions.

40. Draw a square of 2¼″ side and describe about it an octagon without using a circle.

41. A cylinder of 2½ inches diameter, standing erect on one of its ends, is intersected by two planes at right angles to each other and to the vertical plane, the line of intersection of the two cutting planes being 3 inches above the horizontal plane and ½ an inch from the axis of the cylinder, and one of the two planes being inclined at an angle of 60° to the horizontal plane. Draw the plan and elevation of the cylinder assuming the portion above the cutting planes to be removed, and give the true shape of the top of the cylinder after its intersection.

42. On a map a distance known to be 10 miles measures 2⅛″. Construct a diagonal scale to read miles and furlongs, and long enough to take off 15 miles.

43. Construct an irregular pentagon ABCDE from the following data. Sides AB=2¼″, BC=1⅞″, CD=2″, DE=1¾″. Diagonal AD=3″. Angles ABC=120°, CDE=112° 50′.

44. A right hexagonal prism of 1½″ sides and 2½″ height, stands on end with one of its sides parallel to and 1″ distant from the vertical plane. Find the shadow cast by it on both planes

from a luminous point placed 4" above the horizontal and 5" from the vertical plane and six feet distant from the axis of the plane.

45. Construct an ellipse major axis 3¼", minor axis 2¼".

46. A pyramid 2¼" high stands on a square base of 1½ inch side, one of the sides being inclined at an angle of 30° to the ground line and the nearest point being 1" from the vertical plane. The pyramid is intersected by a plane which cuts through its axis at a point 1½" high. This plane is inclined at an angle of 60° to the horizontal plane and 90 to the vertical plane Show the section in plan and elevation, and draw its true shape.

47. Draw the scale whose representative fraction is $\frac{1}{12}$, so that any measurement from an inch to twelve feet may be read off it.

48. Three similar right cones, two inches high, resting on their bases which are one and a half inch in diameter, are symmetrically placed so as to be half an inch distant from one another at their nearest points, and so that two of them are close to and equidistant from the vertical plane. A sphere of two inches diameter is dropped in between the three cones until it comes to a position of rest. Draw the plan and elevation of the group.

49. Draw the shadow cast by light falling at the conventional angle in a semicircular niche two inches wide and four inches high, the top of the niche being round and the lower part flat.

50. Construct a diagonal scale 6 inches long for a Representative Fraction of ^{1} so as to measure feet and inches.

51. A tetrahedron, edges 2 inches long stands with one edge of its base resting on the horizontal plane and at an inclination of 60° to the vertical plane from which its nearest point is one inch distant. The surface of the base, which is turned towards the spectator makes angle of 30° with the horizontal plane. Draw the horizontal and vertical projections.

52. Draw a square of 1½ inch side, then construct a rhomboid an isosceles triangle, and a right angled triangle, each equal in area to the square.

53. Two converging lines meet at a point beyond your table, find by construction the line bisecting the angle between them.

54. On a base of 1½" construct a regular pentagon.

55. Show the horizontal and vertical projection of a line 2 inches long when :—

(*a*) it is parallel to the horizontal plane and inclined at 30° to the vertical plane.

(*b*) it is inclined at 50° to the H.P. and 30° to the V.P.

56. Construct a scale to show yards and feet, the Representative Fraction being $\frac{1}{12}$.

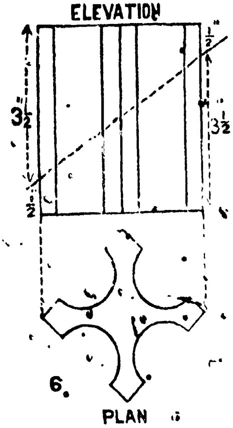
57. A rectangular prism 3 inches long having a base 1½ inches square rests with one edge of its base on the H.P. and has its axis inclined at 30° to the H.P. The edge on which it rests is inclined at 20° to the V.P. Draw its projection on the two planes.

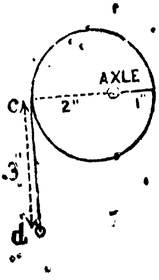
58. A cube 2″ side stands on the H.P. one face being inclined at 30° to the V.P. Draw its shadow.

59. On a map a distance which is known to be 935 yds measures 4¼ inches. Draw the scale so as to read chains and tenth of chains.

60. Three smooth solid spheres, measuring 4, 2, 1 inches in diameter, rest on a plane so that each is in contact with the other two. Draw the plan.

61. Draw the true shape of section along the inclined line of the solid figure roughly illustrated in the margin.





62. A chord cd carrying a small weight d is fastened to the rim of an eccentric wheel at c. This wheel is given half a revolution in the right direction; draw the curve which the point d will make.

63. Construct a regular hexagon of 8 square inches area and project its plan and elevation when lying on a plane whose traces are parallel to the gronnd line and are distant from it 4″ in the horizontal and 3″ in the vertical plane.

64. A circular ring, with a rectangular section of ¼ inch width and ½ inch thickness and with a diameter overall of 4 inches is dropped over the apex of a right hexagonal pyramid of 4 inches height, with a base of 2″ sides, in such a manner as to sit evenly on it. Draw the elevation.

65. Construct an equilateral triangle having an area of 4 sqr inches.

66. A regular hexagonal pyramid of 1½ inches side and 3 inches vertical height is turned over so as to rest on one of its sides with the plan of its axis inclined at an angle of 45° to the vertical plane. A slice is removed from the upper portion of the pyramid by means of a horizontal cut made at a level of 1 inch above the ground plane. Draw the plan and elevation of the lower portion left.

67. Draw the scale of which [illegible] is the representative fraction so that yards and feet can be read.

68. A true pyramid which has a hexagonal base of 1½ inch side and is 4 inches high rests on a table and is titled on one of its sides until it just balances after which it is turned so that the plane of its axis is inclined at 45° to the vertical plane. Draw its plan and elevation.

69. Two semi-cylindrical vaults 12 ft. and 10 ft. diameter respectively, spring from the same horizontal level, and meet at an angle of 30°. Draw the plan and show the method of obtaining their curve of intersection.

70. A sphere of 2″ diameter rests on the ground with its centre c, 1½ inches from the vertical plane. Determine a sphere 1⅜″ diameter to touch the given one at a point P whose plan p is 2⅛″ from the ground line and ½″ to the right of c.

71. Construct a scale of 23 ft. = 1″ to show poles, yards and diagonally feet.

72. The vertical cross section through the apex of a right circular cone is an equilateral triangle, each side of which is 2 inches. A fly starts from the point on the base on the extreme left, walking round the surface of the cone to a point on the right side 7 inches below the vertex. Find geometrically the length of the shortest path the fly can take. Scale [illegible].

73. Draw the lines of interpenetration of a sphere and a cylinder inclined to the ground at 30°; the centre of the sphere lies ½ an inch horizontally without the axis of the cylinder and behind it. Dia. of cylinder = 1¼ inches.

74. Determine the tangent plane touching a sphere of 2″ diameter at a point p on the lower side of the horizontal diameter and on the right of the vertical diameter.

75. When an object is said to be drawn to scale? What scales are used for plans of buildings and what for the plans of sites of buildings?

Draw a diagonal scale of time [illegible] adopted to the motion of a ship which sails at the rate of 5 knots an hour. A knot = 6086·7 ft.

mark on this scale a length corresponding to 2 hours and 39 minutes.

76. Explain and illustrate by a diagram the principle of isometric projection and draw from it the isometric scale 4 ft. = 1″. What is the ratio of reduction of the scale to the natural scale? What is a minor scale and where used?

77. A given line AB = 2½ inches long moves with one end B on the circumference of a given circle of 1″ radius, centre C and the other end A on a straight line passing through C. Find the locus of a point P in AB such that AP = 1½ inches.

78. Give the traces of a plane inclined to both horizontal and vertical planes and the projections of a point above it. Show the projections of the perpendicular from the point to the plane, and mark the point of contact.

79. Draw the plan and elevation of an equilateral triangle 2 inches sides, one of them being at right angles to the vertical plane, and the other inclined at an angle of 30 to the latter.

80. The distance between Calcutta and Naihatee, which is known to be 26 miles, measures 9 inches on a map. Construct a scale for this map, to read miles, furlongs and chains.

81. A sphere of 3 inches diameter is penetrated by a cone of base 3″ diameter and axis 4 inches long. The centre of the sphere is at the middle point of the axis of the cone. Draw the curve of interpenetration when the cone is horizontal.

82. Three equal solid spheres of diameter 1½″ are so placed on the ground that each sphere touches the other two. Another of the same diameter is placed on the top touching all of them. Draw the plan and elevation of the group.

83. Inscribe a parabolic curve in a square whose area is equal to an equilateral triangle of side 5″ and find geometrically the area of the parabola so inscribed.

84. A glass inkstand in the form of a pyramid 3½ inches high has a base 3 inches square. The upper portion at a height of 2 inches above the base is hinged as a lid. Draw the plan and elevation when the lid is set open at an angle of 60 degrees.

I. E. 1909 to 1924.

1. Find the true angle between the HT and VT of a given plane and determine the angle the plane forms with the horizontal plane.

2. The base of a pyramid is rectangular measuring 2″ × 3″, height of axis 4 inches. The pyramid is cut through the middle of its height parallel to the base. Draw in isometric, the frustum of the pyramid when it rests on its smaller end on the ground.

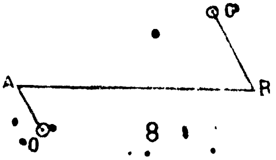
3. Take any two points on the curved surface of a right cone, base 3 inches diameter, height of axis 3 inches. Show the plan and elevation of the shortest line which can be drawn between them.

4. A distance of 1½ miles 3 furlongs is shown on a map by 4½ inches. Draw a scale for the map, showing furlongs by diagonal division.

5. A hemisphere 2½ inches diameter rests with its flat face centrically on the top of a cube of 2 inches side. It is cut by a vertical plane ½ inch distant from the centre of the sphere and making an angle of 45° with the V.P. Draw the sectional elevation of the two solids.

6. Find the shadow cast by a circular cap 3 inches diameter on a vertical square shaft 2 inches diagonal, when the two faces of the shaft are turned at 30° and 60° with the V.P.

7. O and O' are fixed points about which the bars OA, O'B can freely revolve. AB is a coupling bar connecting the free ends of OA and O'B. OA ½ AB and O'B ⅓ AB. Draw the complete locus traced by the centre point of the bar AB.



8. A spiral spring axis vertical is in the form of a square screw thread, side of square ½", external diameter on plan 3", pitch 2¼". Draw the elevation of one complete turn of the spring.

9. On each of the face of a cube 1½" edge stands an equal cube. Make an isometric view of the solid formed by the seven cubes.

10. Draw a scale of miles and furlongs showing diagonally spaces of 10 yards in which 1½ furlongs equal ¼ of an inch.

11. A cone is 5 feet high and has a diameter of 3 feet at the base. Draw this to a scale of 1 inch to the foot and project the surfaces obtained by (a) an oblique section cutting the opposite sides 2 feet from the top and 1 foot from the bottom (b) a vertical section taken 1 foot from centre line.

12. Draw in isometrical projection, a circular iron ring 4 inches in diameter and ¾ inch in section.

13. A hollow cylinder, length 3", outer and inner diameters 1½ and 1" respectively, rests with its axis horizontal, and its ends at an angle of 45° to the vertical plane. It is cut by a plane parallel to the vertical plane of projection through the middle of the axis. Draw the sectional elevation.

14. The horizontal and vertical traces of a plane make angles of 40° and 50° respectively, with the ground line; find a point in the plane, one inch from each plane of projection, and draw a line passing through that point, lying in the plane, and inclined at 30° to the horizontal plane.

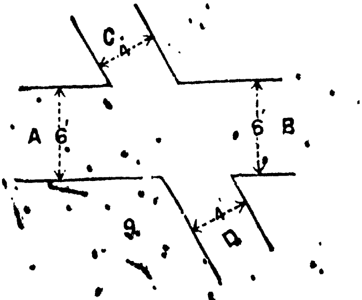
15. A sphere, 2 inches diameter has its centre raised 3 inches above the horizontal plane; the axis of the cylinder enveloping the sphere is inclined 45° to the horizontal plane. Find the line of contact of the two surfaces and the horizontal trace of the cylinder.

16. Represent in isometric projection an octahedron of 1½ inches edge resting with one face on the horizontal plane.

17. A hemisphere 1½ inches diameter rests with its curved surface on the ground, its flat face being horizontal. It is surmounted by a truncated cone, the diameter of base and top 1½″ and ¾″ respectively, and the height 2 inches. Determine the shadow of the solid thus formed on the ground.

18. Draw two lines from a point of the ground line one above it at an angle of 30°, and the other below it at 45° with the G. L. Assume the two lines to be the HT and VT of a given plane. Find the inclinations in degrees of the plane to the two planes of projection and the angle of the plane contained by the two traces.

19. AB and CD are two semicircular archways intersecting each other at 60°. Draw the plan of the intersection of the vaults, assuming that they spring from the same horizontal level.



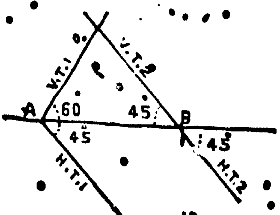
20. Project the shadow cast by a circular flange on a horizontal cylinder.

21. Draw in isometric projection a flight of four steps with 12″ reads and 6″ risers and on one side a 14″ wall with flat horizontal op 18″ higher than the top step and projecting on plan 18″ beyond he bottom step.

22. The traces of a plane are in one straight line and make 35° with XY. Determin the angle which the plane makes with the planes of projection, and show the true angle between the traces of he plane.

Determine a plane inclined to HP and VP at 60° and 52° respectively, and to contain a given point A which is 1½" in front of VP and 2" above HP.

24. Draw the projections of the line of intersection of the two planes shown in the sketch. Make AB 3".



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25. An octagonal slab, whose thicknes is 1", supports a square pyramid (height 2", edge of base 1½") each bottom corner of the pyramid resting on the top corner of the slab. It is placed on the ground in such a way that a diagonal of the base of the pyramid is parallel to the vertical plane, and it is cut by a vertical plane parallel to the VP and passing through the middle of the second faces of the prism on the right and left from its nearest corner. Draw plan and elevation of the portion left.

26. A hollow semi-cylinder 2 feet long is made of zinc sheet 24" × 11". It is placed horizontally with the diameters of its ends vertical. Draw the elevations shewing, first the convex and second the concave face in front by shade lines. Light at the conventional angle. Scale ¼. Are the cast shadows of the semi-cylinder in the two positions, the same or different?

27. Draw the traces of a plane whose inclinations to the horizontal and vertical planes of projection are 55° and 60° respectively.

28. Draw the traces of a plane which shall contain a given point and be parallel to a given plane whose traces do not meet the xy line within the boundary of the drawing paper.

29. A sphere of diameter 3½ inches rests on the ground. Obtain the projections of its section by an oblique plane whose traces are each inclined at 45° to the xy line, the horizontal trace being 1¾ inches from the plan of the centre of the sphere. The plan of the centre of the sphere is 2" from the xy line.

30. A solid is made up of two cylinders as shown in the figure A. Determine the shadow cast by this solid on the planes of projection. Also indicate the parts of the surface of the solid which are in shade.

31. Draw in the isometric projection the box shown in fig. [illegible] Scale half full size. Isometric scale *not* to be used.

32. A hill slope is inclined at 30°, the lines of slope being due East; what is the inclination of a straight road on the hill [illegible] goes in a direction North East? [illegible] 1½

33. Determine a plane inclined to H. P. an[illegible] and 52° and to contain a given point A which is 1½" [illegible] its [illegible] V.P. and 2" above H.P. [illegible]

34. Three places A, B, C on level ground are 4[illegible] feet apart being at the corners of an equilateral triangle. Plot these points to a scale of 1" to 200'. Vertical borings at A, B, C reveal a mineral sea[illegible] depths below the surface of 370', 220', 166'. Find the "dir[illegible] inclination of the strata assuming it to be a plane. Sh[illegible] the "outcrop" of the seam if its plan were continued to the sur[illegible] plane.

35. Draw a cube of 2½' side standing on a squa[illegible] planes of [illegible] traces. and 1' high and on top of the cube place a slab [illegible] high and over that another slab circular in [illegible] ys intersecting diameter. A vertical line contains the cen[illegible] of [illegible] of the vaults, the four pieces forming the column. Draw [illegible] el. in isometric projection. Scale 1" = 1'.

36. The diagram shows a cylinder rising [illegible] erical dome. Determine the curve of intersection of [illegible] Draw to twice the scale of the diagram.

37. An embankment is 30 feet high and [illegible] side slopes 1 to 1 Show a road leading down from the top to the bottom inclined at 20° to the H.P. Is it possible to have a road inclined at 45° to the H.P. on this embankment? If so, what is its length.

38. Find the plan and elevation of the point which is situated in each of the planes shown in fig. 1.

39. It is required to fit a cylindrical steam dome of diameter 3 feet and height [illegible] feet on the top of a cylindrical boiler shell of diameter 5 feet. Draw the curve of intersection of the dome and shell. Axis of shell horizontal. Axis of dome vertical scale 1 [illegible] = 1 in.

40. Determine the shadow cast by the cross and plinth on the H.P. and V.P. Also the shadow cast by the cross on the plinth. Rays of light at conventional angle. Scale full size.

41. abcd represents a black board (6' × 3') on which an equilateral triangle of 1½ feet side is drawn. Draw the plans of the board and the triangle when the plane of the former is inclined at 45° to the vertical and the longer edges ad, bc are horizontal. Scale 1' = 1."

42. The vertical and horizontal traces of a plane make angles of 50° and [illegible]° respectively with the ground line. Determine angles the plane makes with the planes of projection and also the angle between them.

43. Two bricks (10"×5"×3") are arranged as shown in plan and side elevation. Represent the bricks in isometric projection.

2

planes show two circles of 3" and 1" diameters, their centres bein These two circles represent the plan of a right cone and a cyl anding upright on the ground; the height of the cone is 3". ermine the curve of their intersection in elevation when the plane aining the axes of both is inclined at 30° to the vertical plane.

45 Draw plan and elevation of a circle of 1½" diameter with ane vertical and inclined to the vertical plane at 45°

Show in isometrical projection a section of:—

eel girder 12"×5" carrying steel joists 4"×1¾" spaced 4' es flush and secured with lugs and bolts.

with members dissociated.

25. An octa or to a large scale the plan and elevation pyramid (height long, with a hexagonal nut screwed on at one amid resting o the horizontal plane at an angle of 45° with the end in such a vertic to the ver

48 the V co are the plans of three sticks whose lower ends a, b, c are ground and whose upper ends meet at O, The height of O is 10 feet from the ground, assuming their inclinations with the ground determine the lengths of the three sticks. Given ao = 2, bo = 3' and co = 4'.

49. A hemisphere of 3 inches diameter rests with its flat face on the ground. It is cut by a vertical plane AB at an angle of 30° with the V. P and passing ½ an inch away from the centre. Draw its sectional elevation

50. Represent in isometric projection the two successive courses of bricks laid in English bond at the corner of a building. Walls 2 bricks thick. Scale 1/12 full size.

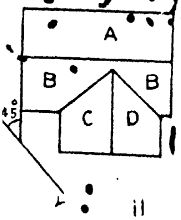
51. It is required to fit a cylindrical steam dome on the top of a cylindrical boiler. Draw the curve of interpenetration of dome and boiler and the development of the dome.

52. Make an isometric projection of a trestle. The top block is triangular edges 4' 6" long and projects 6" beyond the legs at each end. The legs are square in section 3"×3". Vertical height 3'. Assume any suitable inclination for the legs.

53. Draw to a scale of one inch to the foot, the isometric projection of a stone arch of span 9' and rise 3 feet. The curve is a semi-ellipse. The joints divide the curve into nine equal parts. The joints are normal to the ellipse and each 15" long. The arch spans an opening in a brick wall three bricks thick and the stones forming the

arch are single stones flush with both faces of the wall. No brickwork is to be shown.

54. Draw in isometric projection the joint between the principal rafter and tie beam of a roof truss; the timbers separately drawn so as to show the joint. The principal rafter is 5″ × 6″ and the tie beam is 9″ × 5″. The nose of the rafter is 8″ from the end of the tie bar. The tenon of the rafter is 1½″ thick. Scale ¼ full size.



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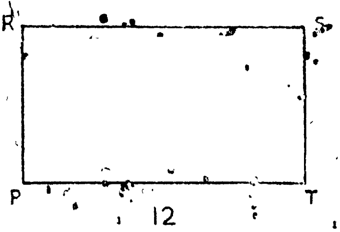
55. The figure 11 shows the plan of a roof truss with gable ends. Scale ⅛″ = 10 ft. Copy the diagram 4½ times the size shown. The roof surfaces A and B are inclined at 40° to the horizontal. What is the inclination of the surface C? Draw a projection of the roof on xy. Set out the true shape and measure the area of the surface B B. Find the angle between the roof surfaces B and C.

56. A spiral (or helical) staircase 3 feet wide winds round a vertical column (or newal) 3 feet diameter. The height of the staircase which makes one complete revolution is 12 feet and there are 20 steps. At the outer edge of the staircase there is a hand rail 3½ feet in height which is attached to the steps by vertical rods, one at the centre of the outer edge of each step. Draw a plan and elevation of the lower half of the staircase with the hand rail and vertical rods represented by lines. Scale 1″ = 1′.

57. The rectangle PRST (fig. 12) is the plan of a building. In order to obtain a large area of roof surface on the side sloping towards PT, it is decided to make the slope of the roof surface at RS = 45° and the slope of each of the remaining three roof surfaces = 30.° Draw the plan, showing the position of the hip rafters and ridge. Scale to be nine times that of the given figure.



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